Underwriting and Ambiguity: 
An Economic Analysis

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Abstract: A simplified financial-economic theory of the insurance firm under uncertainty is used to determine whether ambiguity about the expected claim frequency and/or the claim severity distribution for potential insured losses has any impact on the insurance rate. The model shows that the risk charge for ambiguity in both the claim frequency and claim severity distributions is higher than the risk charge for ambiguity in either the claim frequency or claim severity distribution alone. Our theoretical results are consistent with underwriters’ decisions and provide a more complete analysis of the insurance premium setting process than prior research.

INTRODUCTION

Regulators, underwriters, actuaries, and other parties involved in the premium setting process are interested in the effects of ambiguity on insurance rates because of enduring concerns about the affordability and availability of insurance in certain lines. There has been increasing empirical evidence that one reason the insurance industry has been reluctant to cover a number of risks is the ambiguity associated with either the probability of specific events occurring or the magnitude of the potential consequences or both (Hogarth and Kunreuther, 1989a, 1989b). For example, environmental liability is one such difficult-to-insure loss exposure. Further, economists have long puzzled over why people pay excessive rates for insurance in some cases, but underinsure in others. One need only...
consider earthquake and flood insurance as examples of underinsurance. The concept of ambiguity in underwriting helps explain some of these coverage problems and helps identify cases in which alternatives to traditional insurance markets are likely to develop.

The empirical validity of the expected utility theory has been questioned ever since ambiguity (parameter uncertainty) in subjective probability distributions has been considered. In a situation where coverage was provided against a single risk, Kunreuther (1989) has shown that the equilibrium pure premium based on expected utility for the ambiguous and non-ambiguous cases is identical. Assuming risk neutrality, Hogarth and Kunreuther (1989a, p. 7) have also argued “if losses are known, pure premiums will not be affected by the degree of ambiguity concerning the probability of loss.” However, Hogarth and Kunreuther (1989b) have shown that if a firm with a nonlinear, risk averse utility function is insuring more than one independent risk, then an ambiguous probability measure will lead to a higher premium than a precise probability estimate.

So far, no definite answer has been developed in a microeconomics context to the question of whether an ambiguous probability measure will have an impact on the pure premiums embedded in the rates charged by an insurer. Such a framework will be developed here for ambiguity in the expected claim frequency and the expected claim severity. In addition, it will be shown that the negative impact of claim frequency ambiguity on pure premiums found by Kunreuther (1989) and Hogarth and Kunreuther (1989a) results from the restrictive statistical model they used. When a common actuarial risk model is used, the positive impact of ambiguity on risk charges and premiums is revealed.

In fact, on the basis of a survey of insurance underwriters, Kunreuther, Doherty, Hogarth, and Sprance (1989) (hereafter noted as KDH&S) found that, contrary to their theoretical model, ambiguity about the expected claim frequency does have an impact on the underwriters’ pricing decisions. These empirical results might have led one to believe that expected utility fails to predict the pricing behavior of insurance underwriters. This interesting survey provided the motivation for this paper, which will show theoretically how ambiguity in the expected claim frequency and expected claim severity, when properly modeled, does indeed affect the expected utility based equilibrium pure premium.

The purpose of this paper is twofold. First, a simplified theory of the insurance firm under uncertainty is utilized to determine whether ambiguity (uncertainty) with respect to the expected claim frequency and the expected claim severity for a loss has any impact on the rate (premium). Specifically, the supply-side model of Hogarth and Kunreuther (1989b) is extended to incorporate both supply and demand in the analysis. It is
shown that this expanded model is capable of dealing with ambiguity with respect to both the expected claim frequency and the claim severity (size of loss) simultaneously. Another advantage of this simplified model compared to Hogarth and Kunreuther (1989b) is that it can explicitly determine the factors—such as ambiguity in expected claim frequency, ambiguity in expected loss severity, and demand shifts—that affect risk charges or ambiguity premiums. In addition, it is shown that the risk charge associated with ambiguity in the expected claim frequency is usually higher than the risk charge associated with ambiguity in the expected claim severity. Second, using the analytical results derived from the model based on expected utility theory, an explanation for the empirical results reported by KDH&S is provided. In addition, this paper responds to the following questions raised by KDH&S:

(1) Should one be concerned about the effect of the ambiguity on the insurance underwriter’s decisionmaking process?
(2) Do underwriters respond to the actuaries’ estimates of pure premiums differently when ambiguity about claim frequency and the claim severity (loss size) is present?
(3) What are the factors that are important to underwriters when ambiguity exists?

This paper only investigates whether the results obtained from the expected utility theory under ambiguity are consistent with the underwriter’s decisionmaking process as reflected in the survey by KDH&S. The remainder of this paper is organized as follows. First, an expected utility maximization model for insurance underwriters is developed under the assumption that the demand function for insurance faced by the insurer is fixed. Analytical results are then presented for different scenarios. Second, the results of the theoretical analysis are used to explain some empirical results reported by KDH&S. Third, the assumption about a fixed demand function is relaxed. Lastly, a summary and conclusions are presented.

**PURE PREMIUMS, AMBIGUITY RISK CHARGES, AND EXPECTED UTILITY THEORY**

**Assumptions**

First, underwriters and actuaries are assumed to maximize the expected utility of underwriting profits of their insurance company. Second, it is assumed that the Board of Directors or CEO of the insurance company has specified a risk policy that can be mathematically character-
ized by a risk-averse utility function. Third, while the principal analysis of this paper does not explicitly consider investment income in order to keep the model simple and focus on underwriting, we do suggest below how the model would be extended to incorporate investment income.

Actuaries implicitly determine the distribution of losses or pure premiums according to the probability distributions associated with the claim frequency and claim severity for some type of insurable loss. A simplified probability distribution for losses or pure premiums consistent with the individual risk model from actuarial science (see Bowers et al., 1986) is assumed in this paper.

For analytical purposes, an insurer is assumed to issue \( Q \) standardized insurance contracts for homogeneous exposure units. The total premium or revenue of the insurer is denoted by \( TR \), which is a function of \( Q \), and the unit premium, \( r \), which is also a function of \( Q \)—i.e., \( r + r(Q) \). Moreover, the insurance market is assumed to be monopolistically competitive (see McCabe and Witt, 1980; Tirole, 1988). That is, insurers face a downward-sloping demand curve that is highly elastic but not perfectly elastic. The industry demand curve is assumed to be inelastic. The loss distribution per unit of insurance can be obtained by combining the claim frequency and severity distributions, which are assumed to be independent of each other. The potential size of loss (severity) given a claim is denoted by \( t \), the expected claim frequency (or expected number of losses per policy period) is \( \lambda \), the losses per policy are denoted by \( X \), and the number of claims per period, \( N \), is Poisson-distributed with expectation \( \lambda \). Accordingly, the total possible losses per standard exposure unit are \( X = \sum_{n=0}^{N} t_n \), and the total losses of the firm are \( L \); note that \( L \) is the sum over all \( Q \) policies of the individual \( X \) values, so it is also a function of \( Q \) (i.e., \( L = \sum_{i=1}^{Q} X_i \)). The underwriting profit, \( \Pi \), can therefore be expressed as:

\[
\Pi = TR - L
\]

where \( TR = rQ \) and \( L = L(Q) \). When the expected claim frequency is ambiguous, \( \lambda \) becomes a random variable that we denote by \( \Lambda \). All random variables are denoted by capital letters, except \( t \), which is treated as a random variable in order to minimize confusion with total losses of the firm, \( L \). It can be shown that the mean of the pure premium distribution or
the expected loss per standard exposure unit is the product of the expected claim frequency $E(\Lambda)$ and the expected claim severity $E(\iota)$.

$$E(X) = E(N)E(\iota) = E(\Lambda)E(\iota)$$  \hspace{4cm} (2)

The variance of the loss per standard exposure unit is given as follows:

$$\text{Var}(X) = \text{Var}(N)E(\iota)^2 + \text{Var}(\iota)E(N) = (\text{Var}(\Lambda) + E(\Lambda))E(\iota)^2 + \text{Var}(\iota)E(\Lambda)$$  \hspace{4cm} (3)

where $E(\cdot)$ and $\text{Var}(\cdot)$ are the expectation and variance operators respectively.\(^7\)

Now by using some simple algebraic substitutions,\(^8\) we obtain the expected underwriting profit.

$$E(\Pi) = rQ - QE(\Lambda)E(\iota)$$ and

$$\text{Var}(\Pi) = \text{Var}(L) = Q\text{Var}(X)$$

$$= Q E(\iota)^2 \text{Var}(\Lambda) + Q E(\iota)^2 E(\Lambda) + Q E(\Lambda) \text{Var}(\iota)$$

Maximization of the Expected Utility of Profits

Assuming the insurer faces a given and fixed demand function, the underwriter is postulated to set the rate, $r$, based on the estimated quantity, $Q$, at the beginning of the period before the losses are known in order to maximize the utility of the firm’s underwriting profits, $\Pi$ (see McCabe and Witt, 1980). As an approximation in a mean-variance framework, it is assumed the underwriter selects $Q$ to maximize a certainty equivalent, $U$, which is a linear function of the mean and variance of underwriting profits.\(^9\) Thus, the insurer’s problem is to maximize a certainty-equivalent function $U$ based on the mean and variance of underwriting profits:

$$\text{max } U = E(\Pi) - \frac{k}{2} \text{Var}(\Pi)$$  \hspace{4cm} (4)

where $k$ represents the constant absolute risk-aversion coefficient used by the underwriter and based on a risk policy specified by the Board of Directors or the CEO. Equation (4) has been widely used in the finance literature and can be viewed as a certainty equivalent derived from a second-order approximation to a general utility function.\(^10\) We do not assert that the insurer is risk averse, we only suggest that an underwriting-risk policy can be characterized by a risk-averse utility function for analytical
pricing purposes. After substituting equation (1) into equation (4), one obtains:

\[ \text{max } U = E(TR - L) - \frac{k}{2} \text{Var}(TR - L). \]  

(5)

Next, four cases will be analyzed in order to investigate the effect of ambiguous and non-ambiguous claim severity (losses) (AL and NAL) and ambiguous and non-ambiguous expected claim frequency (ACF and NACF) on insurance premiums. Recall that demand function is assumed to be given and fixed in this section. This assumption will be relaxed later in the paper.

**Case I (NAL and NACF).** In this case, it is assumed that both \( \lambda \) and \( \iota \) are not random variables or ambiguous—i.e., \( \text{Var}(\iota) = 0 \) and \( \text{Var}(\Lambda) = 0 \) in (3). In this nonrandom situation, we let \( E(\Lambda) = \lambda \) and \( E(\iota) = \iota_0 \). This case corresponds to the first question posed in each of the four sets of the questionnaires developed and sent to underwriters by KDH&S (see Appendix 1), except that \( \lambda \) is the expected claim frequency for a Poisson loss event, rather than the expected probability for a Bernoulli-type loss. Utilizing equations (1), (2), and (3), equation (5) in this case yields:

\[ \text{max } U = E(TR - L) - \frac{k}{2} \text{Var}(TR - L) \]

\[ = r Q - Q \lambda \iota_0 - \frac{k}{2} Q \lambda \iota_0^2, \]

where \( \iota_0 \) is a specific value of \( \iota \).

The first-order condition with respect to \( Q \) is

\[ \frac{\partial U}{\partial Q} = MR - \lambda \iota_0 - \lambda \iota_0^2 \text{ or } MR = \lambda \iota_0 + \frac{k}{2} \lambda \iota_0^2 \]  

(6)

where, using chain rule, the marginal revenue \( MR \) is given by

\[ MR = \frac{\partial TR}{\partial Q} = r + Q \left( \frac{\partial r}{\partial Q} \right) \text{ with } \left( \frac{\partial r}{\partial Q} \right) < 0. \]

The above result suggests that the underwriters will want to select the utility-maximizing number of contracts to sell, \( Q \), such that the marginal
revenue is equal to expected loss per exposure unit, \( \lambda_{01} \), plus a risk charge, \( \frac{k}{2} \lambda_{01}^2 \), reflecting the decisionmaker’s (the Board of Directors’) aversion to risk and the fact that there is stochasticity (i.e., risk) present even if the claim frequency and severity are known (the marginal expected loss cost for homogeneous exposure units).

The competitive determination of \( Q \) can be illustrated graphically. In Figure 1, the marginal cost line (MCL) is shown parallel to the horizontal axis. The firm’s demand curve, \( D \), is assumed to be linear, elastic, and downward-sloping in the relevant range in this monopolistically competitive market (see Witt, 1973a). The marginal revenue curve, \( MR \), lies below the demand curve. The utility-maximizing output, \( Q_1 \), and premium rate, \( r_1 \), can then be determined.

**Case II (NAL and ACF).** In this case, size of loss, \( \iota \), is assumed to be non-ambiguous (i.e., \( \text{Var}(\iota) = 0 \)), but the expected claim frequency \( \Lambda \) is ambiguous (i.e., \( \text{Var}(\Lambda) > 0 \)). This case corresponds to the second question in the questionnaire distributed by KDH&S.

Using the results of equations (2) and (3), equation (5) becomes:

\[
\max U = rQ - \iota_0 E(\Lambda) Q - \frac{k}{2} Q \iota_0^2 \text{Var}(\Lambda) - \frac{k}{2} Q \iota_0^2 E(\Lambda). \tag{7}
\]
The first-order condition with respect to Q is

\[
\frac{\partial U}{\partial Q} = MR - E(\Lambda) - \frac{k}{2} t_0^2 \left( \text{Var}(\Lambda) + E(\Lambda) \right) = 0, \text{ or }
\]

\[
MR = t_0 E(\Lambda) + \frac{k}{2} t_0^2 \left( \text{Var}(\Lambda) + E(\Lambda) \right).
\]  

(8)

Equation (8) shows that expected utility is maximized when marginal revenue is equal to expected marginal cost, other things being equal. Marginal cost in this case is the expected loss, \( t_0 E(\Lambda) \), plus a risk charge, \( \frac{k}{2} t_0^2 (\text{Var}(\Lambda) + E(\Lambda)) \), for ambiguous probability. The risk charge for ambiguous probability is a function of the risk-aversion coefficient, \( k \), which is specified by the insurer’s CEO or Board of Directors, the mean and variance of the expected claim frequency distribution, and the size of the loss.

The marginal cost line, MCL2, can be represented graphically, as shown in Figure 1. The vertical intercept is the expected loss plus the risk charge for ambiguous expected claim frequency. It can be seen that \( Q_2 < Q_1 \) and \( r_2 > r_1 \) because MCL2 > MCL1. It should be noted that MCL2 > MCL1 because the risk charge is positive in MCL2. In other words, when the expected claim frequency is ambiguous, the underwriter will want to write fewer policies at a higher premium than in the case where the probability of loss is certain. This result is consistent with the theory of the insurance firm under uncertainty. It should also be noted that this result differs from the theoretical results of Hogarth and Kunreuther (1989) because they used a simplified model that showed that ambiguity in the expected claim frequency had no effect on the premium.

**Case III (AL and NACF).** This case assumes that size of loss, \( t_0 \), is ambiguous or random, while the expected claim frequency is certain (i.e., \( \Lambda = \lambda \) with probability one). To maximize expected utility by using the certainty equivalent, equation (5) now becomes:

\[
\max U = rQ - E(1) \lambda Q = \left( \frac{k}{2} \right) \lambda (\text{Var}(1) + E(1)^2)
\]

\[
\frac{\partial U}{\partial Q} = MR - \lambda E(1) - \left( \frac{k}{2} \right) \lambda (\text{Var}(1) + E(1)^2) = 0, \text{ or }
\]

\[
MR = \lambda E(1) - \left( \frac{k}{2} \right) \lambda (\text{Var}(1) + E(1)^2).
\]  

(9)
Equation (9) states that expected utility is maximized when the premium is equal to the expected loss, $\lambda E(\iota)$, plus a risk charge for loss ambiguity, $\left(\frac{k}{2}\right)\lambda (\text{Var}(\iota) + E(\iota)^2)$.

The utility-maximizing output, $Q_3$, and premium, $r_3$, are shown in Figure 1. It is obvious that $Q_3 < Q_1$ and $r_3 > r_1$, because $\text{MCL}_3 > \text{MCL}_1$. Furthermore, $Q_2 < Q_3 < Q_1$ and $r_2 > r_3 > r_1$, whenever the risk charge in MCL$_2$ is greater than the one in MCL$_3$. MCL$_3$ is less than MCL$_2$—i.e., when $\lambda (\text{Var}(\iota) + E(\iota)^2)$ is less than $t_0^2 (\text{Var}(\Lambda) + E(\Lambda))$. The condition for $\lambda (\text{Var}(\iota) < t_0^2 \text{Var}(\Lambda)$ is $\frac{\text{Var}(\iota)}{t_0^2} < \frac{\text{Var}(\Lambda)}{\lambda}$. Note that $\frac{\text{Var}(\iota)}{t_0}$ and $\frac{\text{Var}(\Lambda)}{\lambda}$ represent the standardized coefficient of variation measures for the severity or size of loss and the expected claim frequency, respectively. If we assume that $\frac{\text{Var}(\iota)}{t_0} = \frac{\text{Var}(\Lambda)}{\lambda}$ then $\frac{\text{Var}(\iota)}{t_0} \left(\frac{1}{t_0}\right) < \frac{\text{Var}(\Lambda)}{\lambda}$ because $\left(\frac{1}{t_0}\right)$ is less than one for non-trivial losses. Therefore, $\text{MCL}_3$ is less than $\text{MCL}_2$, and $r_2 > r_3$.

The above analysis suggests that the underwriters would price insurance coverages with an ambiguous expected claim frequency higher than coverages with ambiguous size of loss if the standardized measures of variation for size of loss, $\frac{\text{Var}(\iota)}{t_0}$, and expected claim frequency, $\frac{\text{Var}(\Lambda)}{\lambda}$, were equal. This result is obtained because the risk charge for expected claim frequency ambiguity is higher than the risk charge for loss ambiguity.

**Case IV (AL and ACF).** For this case, it is assumed that both size of loss, $\iota$, and the expected claim frequency, $\Lambda$, are ambiguous. In this case, equation (5) becomes:

$$\max U = rQ - E(t)E(\Lambda)Q - \frac{k}{2} [\text{Var}(\Lambda)E(\iota)^2 + \text{Var}(\iota)E(\Lambda) + E(\Lambda)E(\iota)^2]$$

The first-order condition is:

$$\frac{\partial U}{\partial Q} = \text{MR} - E(t)E(\Lambda) - \frac{k}{2} [\text{Var}(\Lambda)E(\iota)^2 + \text{Var}(\iota)E(\Lambda) + E(\Lambda)E(\iota)^2] = 0,$$ or

$$\text{MR} = E(t)E(\Lambda) + \frac{k}{2} [\text{Var}(\Lambda)E(\iota)^2 + \text{Var}(\iota)E(\Lambda) + E(\Lambda)E(\iota)^2] \quad (10)$$
Equation (10) shows that the underwriters will try to sell the number of policies such that the premium equals the expected loss plus a risk charge for the intrinsic stochasticity of the model and additional risk charges for ambiguity in both the expected claim frequency and the claim severity. The extension of the preceding model to include investment income is straightforward. The certainty-equivalent utility function, \( U \), becomes

\[
Q(1 + E(i)) - E(\Lambda) - kQr^2 \text{Var}(i) + \text{Var}(\Lambda)E(1)^2 + \frac{1}{2}Qr^2 \text{Var}(i) + \text{Var}(\Lambda)E(1)^2
\]

where \( i \) is the random return on investments, with expectation \( E(i) \) and variance \( \text{Var}(i) \). The first-order condition becomes

\[
MR = r(1 + E(i)) - E(\Lambda) - k2Qr^2 \text{Var}(i) + \text{Var}(\Lambda)E(1)^2 + \frac{1}{2}Qr^2 \text{Var}(i) + \text{Var}(\Lambda)E(1)^2
\]

In Figure 1, MCL4 is greater than either MCL2 or MCL3 because of a higher risk charge. Therefore, one obtains \( Q_4 < Q_2 < Q_3 < Q_1 \) and \( r_4 < r_2 < r_3 > r_1 \), when the standardized measures of variation for the expected claim frequency and the size of loss are equal. Furthermore, the risk charge for ambiguity in both expected claim frequency and claim severity is higher than the risk charge for either the ambiguous expected claim frequency or ambiguous claim severity case alone.

However, if the insurer were risk neutral (\( k = 0 \)), then all ambiguity risk charges would be zero. The premium would not be affected by ambiguity in the expected claim frequency or the size of loss if the insurer exhibited a risk neutral policy. Such a premium is frequently referred to as an actuarially “fair” premium in the financial-economic literature. This result is consistent with both modern insurance and finance theory. The results for the four cases examined above are summarized in Table 1 below.

**THEORETICAL RESULTS AND EMPIRICAL EVIDENCE**

In this section, the theoretical results derived in the preceding section will be compared with empirical evidence provided by KDH&S. Only the first three scenarios of the four scenarios from KDH&S will be investigated because the model probably is not appropriate to analyze the earthquake
scenario utilized in KDH&S. There are four cases in each of the three scenarios analyzed. Thus, twelve cases will be evaluated here.

**Table 1. Summary of Theoretical Results**

<table>
<thead>
<tr>
<th>Case</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. NAL and NACF</td>
<td>$MR = \mu_0 + \frac{k}{2} \lambda^2 \mu_0^2$</td>
</tr>
<tr>
<td>2. NAL and ACF</td>
<td>$MR = \mu_0 E(\Lambda) + \left(\frac{k}{2}\right) \lambda (\text{Var}(\Lambda) + E(\Lambda)^2)$</td>
</tr>
<tr>
<td>3. AL and NACF</td>
<td>$MR = \lambda E(t) + \left(\frac{k}{2}\right) \lambda (\text{Var}(t) + E(t)^2)$</td>
</tr>
<tr>
<td>4. AL and ACF</td>
<td>$MR = E(t) E(\Lambda) + \frac{k}{2} [\text{Var}(\Lambda) E(t)^2 + \text{Var}(t) E(\Lambda) E(t)^2]$</td>
</tr>
</tbody>
</table>

**Context-Free Scenario 1 (Potential Loss = $1 Million and Probability of Loss = 5/1000)**

*Case I (NAL and NACF).* The theoretical result for Case I in Section II predicts that the pure premium should be equal to the expected loss ($5000) plus a risk charge for the intrinsic stochasticity (here compound Poisson) inherent in the risk transfer. The empirical evidence provided by KDH&S (see first bar in Figure 2) shows that the premium would have been set at $6,100. The theoretical result could be viewed as being consistent with the evidence because the difference of $1,100 could be considered as a risk charge plus an expense loading that underwriters added to the expected pure premium of $5000.16

*Case II (NAL and ACF).* The theoretical result in the previous section suggested that the premium here should be greater than the one in Case I above because of the risk charge for ambiguous claim frequency. This result is also consistent with the empirical evidence developed by KDH&S (see second bar from left in Figure 2).17

*Case III (AL and NACF).* The theoretical result for Case III in Section II predicted that premiums for Case III should be greater than those in Case I, and possibly less than those for Case II, when the risk charge for ambiguous expected claim frequency is higher than the loss-ambiguity risk charge. Under these conditions, the theoretical result is consistent with the
empirical evidence (see second and third bars in Figure 2) presented by KDH&S.

Case IV (AL and ACF). The theoretical result predicted that the premium, the expected loss plus an expense loading and a risk charge, should be the highest for the case where there is ambiguity in both losses and in expected claim frequency. The empirical evidence on gross premiums, which include expenses (see fourth bar), shows the premium is highest in this case. Thus, the theoretical and empirical results are consistent.

Hazardous-Waste Scenario (Potential Loss = $1 Million and Probability of Loss = 5/1000)

The four questions in this scenario posed by KDH&S were identical to the questions in the first context-free scenario except that a hazardous-waste exposure is added for context purposes. The ranking of the various premiums with different risk charges under the four cases was preserved. Therefore, the theoretical predictions are consistent with the empirical evidence (see Figure 3). However, there are some results that merit some elaboration.

Case I (NAL and NACF). The empirical evidence developed by KDH&S (see the first bar in Figure 3) shows that the premiums underwriters would charge in this hazardous-waste scenario are much higher than those in the context-free scenario 1. Does this mean the expected utility theory failed? We do not think so. One possible explanation is that the underwriters rely more on their intuition and experience rather than actuarial assumptions and predictions in this scenario. Specifically, rational underwriters will probably not believe that there is any realistic case of non-ambiguous losses or probability when they underwrite hazardous-waste coverages, especially if they follow recent court decisions. In the
context-free scenario 1, the underwriters may subjectively associate the relatively precise size of loss and expected claim frequency (Case I, context-free scenario 1) with some insurance coverage such as a basic property insurance coverage for which there is a very stable loss size based on actuarial data collected over time. Therefore, the premiums that underwriters would like to charge in this case could still be consistent with the results predicted by the expected utility theory. However, when the hazardous-waste exposure is added for context to the case, underwriters might not have been comfortable with the assumptions specified by KDH&S that there was no ambiguity with respect to the expected claim frequency and size of loss. In other words, it is possible that the underwriters, on the basis of their experience with increasing frequency and severity of environmental liability losses prior to the survey, decided to add some ambiguity risk charges to the expected losses. Therefore, the premiums they would like to charge under this scenario are higher than the ones in the context-free scenario 1.

An alternative explanation is that the risk-aversion coefficient \( k \) may change when the scenario becomes one involving hazardous waste. Since current estimates of cleanup costs under superfund are about $1 trillion, and the surplus of the entire US property and casualty industry was only $230 billion in 1995, even if the insurance industry had to pay as little as 25% of the cleanup costs, it would bankrupt the entire property and casualty industry. Accordingly, underwriters may become more risk averse under this scenario (see Brockett, Golden, and Aird, 1990).

A third possibility is that the underwriters may not believe that the potential loss is only $1 million for a hazardous-waste case. If the underwriters believe that potential loss is greater than $1 million, they could also
subjectively increase the size of the expected loss. A rational and competent underwriter’s job is to deviate from purely statistical recommendations when such recommendations do not seem to be justified or realistic (otherwise there would be little need for underwriters in commercial liability insurance). Rate making in this area is probably an art as much as a science when one recognizes the evolving nature of the tort liability system in the United States. The socio-legal risk associated with this system probably makes most underwriters very conservative in their projections.

**Case II (NAL and ACF).** The evidence (see second bars, Figures 2 and 3) shows that the average premiums for this scenario are higher than those underwriters would charge in the context-free scenario 1. To determine whether the experimental evidence is consistent with the theoretical results, one needs to refer to equation (8), which shows that the risk charge for ambiguous expected claim frequency is \( \frac{k}{2} \Delta \text{Var}(\Lambda) \).

It seems reasonable to assume that rational underwriters would consider \( \text{Var}(\Lambda) \) to be much higher in the hazardous-waste scenario than in the context-free scenario 1. Underwriters may consider the context-free scenario 1 to be an average of all possible coverages, while the hazardous-waste scenario is a special well-known case of high risk, especially after one recognizes all of coverage disputes in this area. Since the risk charge is a function of \( \text{Var}(\Lambda) \), the premium charged would then be higher. Therefore, this may explain why the premium and the expected-claim-frequency-ambiguity risk charge for the hazardous-waste scenario are higher than the ones in the context-free scenario 1.

A graphic presentation, as shown in Figure 4, may help to provide some additional insight. MCL\(_2^1\) and MCL\(_2^2\) represent marginal cost lines for Case II involving the hazardous-waste scenario and context-free scenario 1, respectively. As shown, \( r_2^1 \geq r_2^2 \) in order to illustrate that the premium and associated ambiguity risk charge for the expected claim frequency in the hazardous-waste scenario is higher. Furthermore, if the underwriters believe that the potential loss is more than $1 million, as suggested above in Case I, then both the expected loss and ambiguity risk charge would be higher, and \( r_2^2 \) would even be higher.

**Case III (AL and NACF).** To determine whether the risk charge for loss-ambiguity is higher in the hazardous-waste scenario than in the context-free one, equation (9) and an analysis similar to Case II can be utilized. The result suggests that the loss-ambiguity premium would be higher in this scenario because \( \text{Var}(\iota) \) is higher than in the context-free scenario 1. However, the experimental empirical evidence (Figures 2 and 3) seems to suggest this may not be the case. That is, the risk-charge for loss-ambiguity
is $1,200 for this scenario while it is $1,700 for the context-free scenario. The inconsistency may result from the already high premiums in Case I of this scenario (NAL and NACF). As suggested for Case I of this scenario, the underwriters surveyed apparently did not believe the non-ambiguous loss and non-ambiguous expected claim frequency postulated for the hazardous-waste scenario. Therefore, they probably subjectively added an ambiguity risk charge in Case I.

Another way to look at this situation is by observing that underwriters do not develop the premiums of Case I and Case II (see first and third bar of Figure 3) of this scenario differently. Note that both cases assume non-ambiguous probability. Therefore, only loss ambiguity matters. There are two possible explanations. It appears that the underwriters either try to add ambiguity risk charges to Case I, as suggested above, or do not believe they should add the ambiguity risk charges to Case III because the loss-ambiguity risk is very small (first and third bar of Figure 3). The first explanation seems to be more reasonable.

Case IV (AL and ACF). Employing equation (10) and an analysis similar to Cases II and III, one can show that the risk charge for ambiguity in the expected claim severity and claim severity is higher for the hazardous-waste scenario than for context-free scenario I. The evidence in Figures
2 and 3 (see the fourth bar in each figure) confirms this theoretical expectation.

**Context-Free Scenario 2**

This scenario is the same as context-free scenario 1 except the magnitude of loss size is increased to $10 million from $1 million. For the first two scenarios evaluated above, the analyses were performed under the assumption that the demand curve was fixed and would not shift. It may be unrealistic to assume that the demand curve will not shift under this scenario. Therefore, the discussion of the context-free scenario 2 will be delayed until the next section, where the fixed demand curve assumption is relaxed.

**RESULTS WHEN THE DEMAND CURVE Shifts**

In Section II, the demand curve was assumed to be fixed in all of the cases and all of the scenarios. However, the demand curve may shift because of ambiguous expected claim frequency or ambiguous loss size, or it may shift because of the dramatically larger size of loss. The assumption that the demand curve is fixed is relaxed in this section.

**Context-Free Scenario 1**

It is possible the demand curve could shift upward and to the right when the expected claim frequency and/or size of the loss are ambiguous, because individual policyholders might be willing to pay higher risk charges and premiums because of the greater uncertainty associated with such ambiguity. Figure 5 shows the utility-maximizing output, $Q^*_2$, and premium, $r^*_2$, for Case II after the demand curve shifts. It can be seen that $r^*_2 > r_2 > r_1$; therefore, the original result in Section II, which states $r_2 > r_1$, is not altered when the fixed demand curve assumption is relaxed. The original results as related to premiums in Cases III and IV will not be altered for similar reasons. However, one interesting result shown in Figure 5 is that $Q^*_2 > Q_2$. Thus, it is also possible that $Q^*_2 > Q_1$ when the fixed demand curve assumption is relaxed. Whether $Q^*_2$ is greater than $Q_1$ will obviously depend on how much the demand curve shifts to the right and how much policyholders are willing to pay for a risk charge in order to eliminate the uncertainty associated with ambiguity in the probability and/or severity of loss.
Hazardous-Waste Scenario

Case II (NAL and ACF). When one compares Case II in this scenario with Case II in the context-free scenario 1, the demand curve may also shift to the right (see Figure 4) for reasons cited above. The policyholders may be willing to pay higher premiums for the same amount of insurance, and the number of policies sold may not change much because they perceive the hazardous-waste scenario to be riskier—that is, \( \text{Var}(\Lambda) \) is greater in this case.

In Case II, the hazardous-waste scenario in Section III, we have shown that the risk charge for ambiguous claim frequency is higher in the hazardous-waste scenario than for the context-free scenario 1. Figure 6 shows that \( r_2^\ast > r_2 > r_2 \) when the demand curve shifts to the right. Therefore, the original result \( (r'_2 > r_2) \) in Case II, the hazardous-waste scenario in Figure 4, is not altered when the assumption of a fixed demand curve is relaxed.

Case III (AL and NACF) and IV (AL and ACF). The basic results obtained for loss-ambiguity and loss-and-frequency-ambiguity under hazardous-waste scenario in Section III would not be altered by using similar analysis.
Context-Free Scenario 2 (Potential Loss = $10 Million and Probability of Loss = 5/1000)

The demand curve will shift to the right if the size of loss increases from $1 million to $10 million, other things being equal.

**Case I (NAL and NACF).** Utilizing equation (8), one can see the expected loss will be higher under this scenario because of the higher potential loss than under the context-free scenario 1. The evidence (first bars of Figures 2 and 7, respectively) is consistent with the results, as expected.

**Case II (NAL and ACF).** The risk charge for ambiguity in claim frequency ($79.3 million) under this scenario is much higher than the risk charge ($2.5 million) under scenario 1, as would be expected. Referring to equation (8), one can see MCL\(_2^*\) is higher in this scenario because of higher potential losses. Furthermore, the demand curve will shift from D to D*. Therefore, the expected claim frequency-ambiguity risk charge is higher when the size of loss is higher, because \((r_{2}^* - 50,000)\) can be seen to be greater than \((r_{2} - 50,000)\), as shown in Figure 8, other things being equal.

**Case III (AL and NACF) and IV (AL and ACF).** Similar analyses can be performed to show that the risk charges for loss-ambiguity and loss-and-
frequency-ambiguity are higher in this scenario than in scenario 1. Again the evidence is consistent with the theoretical expectations or results.

**SUMMARY AND CONCLUSION**

In this paper, it was assumed that the underwriters would select an output such that the characteristic function for the utility of the firms' underwriting profits specified by the CEO or Board of Directors would be maximized. Using expected utility theory, various utility-maximizing premiums and outputs based on certainty equivalents were obtained under various assumptions about the expected loss frequency and size of loss in terms of ambiguity. Some of the main results suggested are summarized below.

First, the underwriter will charge higher risk charges and premiums for ambiguous claim frequency and/or ambiguous size of loss, other things being equal. Second, the risk charge for frequency-and-loss-ambiguity is higher than either the risk charge for ambiguity in the expected claim frequency or claim severity alone. Moreover, the risk charge for frequency-ambiguity is higher than the risk charge for loss-ambiguity in the three scenarios discussed in this paper, when the standardized measures of variation for the expected claim frequency and the size of loss are equal. Third, the risk charges for frequency-ambiguity, severity-ambiguity, and frequency-and-severity-ambiguity are higher in the hazardous-waste scenario than in the context-free scenario 1. Fourth, all of the analytical results are consistent with the empirical evidence provided by KDH&S.

Fig. 7. Context-free scenario 2. Potential loss = $10 million.
Our results differ from those obtained in other studies because we assume the underwriters will select an output to maximize utility, whereas the earlier studies cited in the paper assume that underwriters would charge a premium that uses the expected loss to maximize utility. In addition, we used a Poisson model for claim frequency rather than a Bernoulli loss/no loss model. This reflects the fact that the policy does not terminate when a loss occurs, but rather there is a small chance that several claims might arise during a single policy period. (This has proven to be especially important in environmental liability insurance.) We believe our approach and assumptions are more realistic. Since underwriters are the employees of the insurance firm, their behavior should follow and be consistent with the economic theory of the insurance firm under uncertainty. Furthermore, empirical evidence about the underwriting process appears to support the predictions of the theoretical model in this paper.

It should also be emphasized that our model also considers the demand function faced by insurers, while Hogarth and Kunreuther (1989b) used only a supply-side model. In addition, while we have considered that underwriting process in a microeconomic context with the possibility of multiple policies being sold (i.e., Q is a variable of interest), the questionnaire in KDH&S might be construed as involving a single policy with no opportunity for pricing so as to maximize profit. Within this “single hazard” context, our compound model with claim frequency modeled as a

![Fig. 8. Context-free scenario 2 with a shift in demand.](image-url)
mixed Poisson distribution can be applied in a manner similar to the Bernoulli model used by Kunreuther (1989), Hogarth and Kunreuther (1989a), and KDH&S. Here we find a definite effect of ambiguity in either expected claim frequency (i.e., \( \text{Var}(\Lambda) > 0 \)) or in claim severity (i.e., \( \text{Var}(\iota) > 0 \)), unlike the results in Kunreuther (1989) and Hogarth and Kunreuther (1989a), who predict only ambiguity in claim severity should matter. Unlike the theoretical model in Hogarth and Kunreuther (1989), empirical evidence is consistent with the implications of our theoretical model.

This study has shown that expected utility theory can explain the empirical fact that ambiguity in claim frequency affects premiums when modeled appropriately. This study was also able to respond to three of the questions posed by KDH&S. First, one should be concerned about the effect of ambiguity in claim frequency and claim severity on the decisionmaking process of insurance underwriters. Second, both our theoretical results and survey evidence suggest that underwriters interpret and respond to actuarial estimates of pure premiums differently when ambiguity about loss size and/or claim frequency is present. Finally, some of the factors important to underwriters may include risk aversion based on insurer policy, the frequency of losses, and related parameter uncertainty, the potential size of loss, and any ambiguity associated with it. Clearly, factors other than ambiguity affect choice under uncertainty. These include problems such as adverse selection, fear of bankruptcy, and moral hazard as well as possible effects due to framing (see Tversky and Kahneman, 1981) and other psychological influences. Examining the importance of these other factors might extend the current research.

NOTES

1 See Machina (1987) and Bowers et al. (1986).
2 For a detailed review of the literature, see Kunreuther and Hogarth (1992).
3 KDH&S use the term “ambiguity premium” in their paper, whereas we use the term “risk charge” to minimize confusion with respect to other uses of the term “premium” in the insurance literature. Ambiguity premiums should not be confused with pure premiums, net premiums, or gross premiums. Ambiguity premiums are used in the context of a risk charge associated with the ambiguity risk. For example, the probability-ambiguity risk premium refers to the risk charge needed to be included in the premium due to ambiguous probability in addition to the expected loss. Pure premiums are used in the context of the price charged by insurers to cover expected losses. Since expenses are not included in the distribution of losses, this explains the reference to a pure premium distribution in insurance literature. The net rate will be defined to be the expected loss or pure premium plus the risk charge. The rate or gross rate includes the net rate plus an expense loading.
4 Our model is primarily applicable to property and liability (p/l) insurance, since ambiguity is more problematic in some p/l lines of insurance, such as environmental liability and products liability. However, the model could be applied to cases of ambiguity in life and health loss exposures.
For an alternative rationale, see Greenwald and Stiglitz (1990).

Generally, actuaries make a recommendation on the expected pure premium for some insurance coverage to the underwriters. It is assumed that the underwriters use such a recommendation as a base for determining the risk charge and gross rate to charge. However, as Kunreuther (1989) has noted, underwriters probably tend to rely more on their own experience and intuition than on actuarial data and estimates. This is especially true for high-limit, difficult-to-insure liability exposures.

For a discussion of equations (2) and (3), see Bowers et al. (1986) and Witt (1973b).

Substituting \( \sum X_i \) for \( L \) in equation (1) and taking expectations yields, \( E(\Pi) = rQ - QE(X) \).

Then using equation (2), substitute \( E(\lambda)E(\iota) \) for \( E(X) \) to obtain \( E(\Pi) \). Recall that \( \text{Var}(\Pi) = Q\text{Var}(X) \) since \( X_i \)s are independent and identically distributed. Then substitute equation (3) for \( \text{Var}(X) \) to obtain \( \text{Var}(\Pi) \).

If we assume a monotone relationship between the price of the policy and the quantity of policies sold, then the underwriter’s objective function could alternatively be written in terms selecting the premium rate, so as to maximize expected utility. For simplicity throughout, we shall use the quantity selection formulation as in equation (4).


Hogarth and Kunreuther (1989b) and MacMinn and Witt (1987) also use a risk-averse utility function.

The above result is based on the assumption that the demand function is fixed and known by the underwriter. The relationship between \( Q_2 \) and \( Q_1 \) could change if the assumption about a fixed and known demand function were relaxed.

See MacMinn and Witt (1987), and McCabe and Witt (1980).

For simplicity, we assume that investment returns are not correlated with underwriting results. MacMinn and Witt (1987) also assumed that investment income and underwriting profits are independent. Cummins and Harrington (1987) provide empirical evidence consistent with that assumption. If investment returns and underwriting profits are correlated, the risk premium will be higher (lower) in the case of negative (positive) correlation. If investment returns and pure risk exposures are not correlated, then the addition of investment risk increases the risk-premium, other things being equal.

It should be noted that these results are based on a fixed expense loading in the model. The same results would be obtained if expenses were variable but not random. However, the algebra becomes more complicated.

It should be noted that the questionnaire designed by KDH&S is not clear on whether the “minimum premium” refers to a pure premium or gross premium (see Appendix 1). However, the authors of this study believe that most of the underwriters would consider “minimum premium” as a gross premium in practice. That is, they would include an expense loading on top of the pure premium.

Figures 2, 3, and 7 are reproduced from KDH&S, except we refer to expected claim frequency rather than expected probability loss in the relabeled figures.

REFERENCES


APPENDIX 1*

Suppose you are faced with four different situations where you are given some information on which to base an annual premium for a risk. No other information is available and no new information will become available before setting the premium.

1. In situation one you learn that:
   Estimates of possible insured losses: All experts agree that if losses occur they will equal $1 million.
   Estimates of annual probability of losses: All experts agree on 5 in 1000.
   Question: What is the minimum premium you would charge to accept this risk?

2. In situation two you learn that:
   Estimates of possible insured losses: All experts agree that if losses occur they will equal $1 million.
   Estimates of annual probability of losses: 5 in 1000, however, there is wide disagreement on this figure and a high degree of uncertainty among the experts.
   Question: What is the minimum premium you would charge to accept this risk?

3. In situation three you learn that:
   Estimates of possible insured losses: Experts’ best estimate is that if losses occur they will equal $1 million. However, estimates range from negligible to $2 million.
   Estimates of annual probability of losses: All experts agree on 5 in 1000.
   Question: What is the minimum premium you would charge to accept this risk?

4. In situation four you learn that:
   Estimates of possible insured losses: Experts’ best estimate is that if losses occur they will equal $1 million. However, estimates range from negligible to $2 million.
   Estimates of annual probability of losses: 5 in 1000; however, there is wide disagreement on this figure and a high degree of uncertainty among the experts.
   Question: What is the minimum premium you would charge to accept this risk?

* This is a reproduction of Table 2 from KDH&S (1989).
APPENDIX 2

Derivation of equations (2) and (3)

Derivation of equation (2):
A1.1 \( E(X) = E[E[X \mid N]] \) property of iterated conditional expectation
A1.2 \( = E[NE[t]] \)
A1.3 \( = E(N)E(\iota) \)
A1.4 \( = E(\Lambda)E(\iota) \) \( E(N) = \lambda \), which equals \( E(\Lambda) \) when \( \lambda \) is not known, i.e., ambiguous claim frequency

Derivation of equation (3):
A2.1 \( \text{Var}(X) = E(X^2) - [E(X)]^2 \) definition of variance
A2.2 \( = E[E(X^2 \mid N)] - [E[E(X \mid N)]]^2 \) property of iterated conditional expectation
A2.3 \( = E[E(X^2 \mid N)] - [E[E(X \mid N)]]^2 + E[[E(X \mid N)]^2] - E[[E(X \mid N)]^2] \)
A2.4 \( = E[\text{Var}(X \mid N)] + \text{Var}[E(X \mid N)] \)
A2.5 \( = E[N\text{Var}(t)] + \text{Var}[NE(t)] \)
A2.6 \( = E(N)\text{Var}(t) + [E(t)]^2\text{Var}(N) \)
A2.7 \( = E(\Lambda)\text{Var}(t) + [E(t)]^2[\text{Var}(\Lambda) + E(\Lambda)], \) which was to be shown

Note that \( \text{Var}(N) = \text{Var}(\Lambda) + E(\Lambda) \) since \( \text{Var}(N) = \text{Var}[E(N \mid \Lambda)] + E[\text{Var}(N \mid \Lambda)] \) (see 2.4 above) and \( E(N \mid \Lambda) = \Lambda \) and \( \text{Var}(N \mid \Lambda) = \Lambda \) for \( N \sim \text{Poisson} (\lambda) \) and \( \lambda \) is not known, i.e., ambiguous.