Further Considerations of Underwriting Margins, Interest Rates, Stability, Stationarity, Cointegration, and Time Trends

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Abstract: This article provides a philosophical discussion detailing the limitations of univariate analysis in the pre-testing step of data analysis. The case in point is the relationship between the property-liability aggregate underwriting margin and interest rates. Haley (1993) and Choi, Hardigree, and Thistle (2002) both found strong evidence indicating such a relationship exists. Since then assorted authors, relying extensively on univariate analysis, have questioned the cointegration conclusion. The paper uses a cointegration analysis of the property-liability aggregate underwriting margin and interest rates (1930–2000) to illustrate the discussion. [Key words: cycles, cointegration, underwriting margins.]

The importance of interest rates is well established in insurance pricing theory.¹ Since insurance premium inflows and claim cost outflows occur at different points in time, interest rates and insurance prices have a (theoretically) negative relationship. Haley (1993) and Choi, Hardigree, and Thistle (2002) both conclude that a negative cointegrating relationship does indeed exist between the underwriting margin and the risk-free rate.² Some researchers, however (Leng et al., 2002; Harrington and Yu, 2003; Leng, 2006a, 2006b; Leng and Meier, 2006), have questioned these cointegration conclusions, mostly basing their contrary arguments on the results of univariate analysis. Leng and Meier (2006) comment, at the end of their article, that the theoretical relationship between the loss ratio and interest rates (in the U.S.) is “not well supported empirically.”

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The purpose of this article is to present a philosophical perspective on the limitations of using univariate analysis as a pre-test while evaluating the relationship between underwriting margins and interest rates. My perspective is concerned with careful interpretation and extraction of the information contained in the underwriting margin and interest rate time series. My arguments consider what a priori restrictions should be placed on the data and what the assorted univariate statistical methods are capable of describing. The context of this article includes a thoughtful and back-to-the-basics consideration of equilibrium conditions and stationarity. A cointegration analysis of the property-liability aggregate underwriting margin and the 90-day Treasury bill rate is presented, in part, to illustrate the discussion. The analysis also provides evidence in support of insurance pricing theory.

**Equilibrium**

Economic equilibrium can be broadly defined as a set of circumstances that possesses a tendency for no change. Equilibrium conditions are considered stable if, when a deviation occurs, there is a tendency to return to them. Since many economic systems (or subsystems) are subject to frequent shocks from a wide variety of sources, the actual (exact) attainment of equilibrium is probably quite rare. What is more generally the case in a stable equilibrium system is a continuous effort to move towards a target, yet rarely settling on the target. A stationary time series (fixed mean and variance) can, for instance, be indicative of equilibrium conditions. A multivariate set of circumstances that is indicative of equilibrium conditions occurs when two or more variables have the collective tendency to move towards an attracting space. In the two-variable case this attractor is a line.

Two major variables of interest in an equilibrium analysis of the property-liability underwriting cycle are the underwriting margin and the risk-free interest rate. As already mentioned, the theoretical insurance pricing models promote the expectation of a negative relationship between these two variables with variations in the risk-free interest rate being an explanatory factor for variations in the underwriting margin. Figure 1 displays the 90-day Treasury bill rate for new issues and the aggregate underwriting margin for stock property-liability companies, 1930–2000. Visually, both variables appear to be nonstationary since they do not fluctuate around a fixed mean (for the entire period). The interest rate variable trends upward until 1981, after which it begins to decline, while the underwriting margin trends downward until approximately 1984, after which it begins a slight upward trend.
Cointegration Analysis

The cointegration technique for analyzing long-run equilibrium relationships in time series data was first developed by Engle and Granger (1987). According to this technique, two nonstationary variables are cointegrated if (1) they can be linearly combined \((x_t - \beta y_t)\) to create a third variable that is stationary and (2) this resulting new variable has a statistically significant relationship with the first differences of at least one of the original variables.

Cointegrated variables are interpreted as having a long-run equilibrium relationship. The \((x_t - \beta y_t)\) relationship defines the long-run attractor line for the variables, while the stationarity of the deviations from this line displays the short-run dynamics and indicates a tendency for the relationship to return to the line. Rewriting the expression for this relationship gives\(^5\)

\[
x_t = \beta y_t + \varepsilon_t
\]

where \(\varepsilon \sim N(0, \sigma^2_\varepsilon)\). \(E(\varepsilon) = 0\) is an important property indicating that the two-variable system is attempting to achieve equilibrium.

A prerequisite to any conclusion of cointegration is, of course, that the underwriting margin and the interest rate variables both be nonstationary. It is this “pre-test” analysis that is the main source of differing opinions and conclusions by other researchers.

Pre-Testing and Stationarity

A stationary time series, \(x_t\) variable has three conditions (Vandaele, 1983):

\[
E(x_t) = \mu \text{ (fixed mean)} \tag{2a}
\]

\[
E[(x_t - \mu)^2] = \sigma^2 \text{ (fixed variance)} \tag{2b}
\]

\[
(E[(x_t - \mu)(x_s - \mu)])/\sigma^2 = \rho_{t-s} \text{ (fixed autocorrelation coefficients).} \tag{2c}
\]

Each condition refers to a particular feature of the variable remaining constant through time. Of greatest significance to a cointegration study is a fixed mean. The fixed variance is important, but can be relaxed under certain conditions.\(^6\) Neither variable presented in Figure 1 appears to fluctuate around a fixed mean over the entire time period.
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The source of mean nonstationarity in the underwriting margin is the essential issue in Harrington and Yu (2003). Their results clearly show mean nonstationarity (1953–1998) and they attribute the nonstationarity to a deterministic time trend, thereby asserting (univariately) a process onto the time series. It is important to mention, however, that nonstationarity is a much more open-ended property than stationarity. Nonstationarity, as the word implies, is simply the absence of stationarity. The sources of nonstationarity in a time series variable may be few or myriad. A cointegration analysis is concerned with mean stationarity and, in this case, the possible long-run relationship between the risk-free interest rate and the underwriting margin. I am not concerned with a full (univariate) determination of these variables’ data generating processes (DGP). The Newbold, Leybourne, and Wohar (2001) analysis of U.S. real GNP concludes that a single notion of stationarity (trend- or difference-stationary) does not apply during 1875–1993. They are not surprised by this conclusion (neither am I) and continue on to state that “stationarity of any sort” is "a priori implausible" for the entire 119 years.

The fact that nonstationarity has a wide variety of forms is reflected by the considerable amount of literature that has been devoted to developing tests for it. According to Robinson (1994), the literature’s emphasis on unit roots may have obscured the fact that a unit root is “an extremely specialized form of nonstationarity.”

Fig. 1. Ninety-day Treasury bill and property-liability underwriting margin for stock companies, 1930–2000.
Instead of being enamored with the specificity of unit root tests, Granger (1995, 1997) describes time series variables as possessing the property of “persistence” or “extended-memory.” A time series process is said to have extended memory if the conditional (univariate) expected mean over a large time horizon does not tend to be constant—i.e., has no fixed mean.

Pre-Testing and Structural Stability

Several articles (Leng et al., 2002; Leng and Meier, 2006; Leng, 2006a, 2006b) follow a structural stability line of thought in evaluating insurers’ aggregate underwriting results. While this type of analysis can provide important insights into certain aspects of a time series, it is not needed as a cointegration pre-test (nor does it affect the cointegration test). For example, through a series of hypotheses, Leng and Meier (2006) conclude that the U.S. property-liability loss ratio (1955–1997) is nonstationary. They then proceed to determine that a structural break occurred around 1986 and divide the time series into pre-break and post-break subperiods. Subsequently, they conclude that the interest rate and loss ratio are not cointegrated in the post-break period. However, dividing the data (univariately) into pre-break and post-break subperiods interferes with a multivariate long-run analysis. In a similar fashion, Leng (2006a, 2006b) eliminates data prior to 1952 with the reasoning that World War II events and price controls and the passage of the McCarran-Ferguson Act may have created abnormal behavior. In the cointegrating equilibrium analysis that follows, unusual fluctuations during the mid-1940s are easily identified, but the industry’s effort to maintain equilibrium is clear. Eliminating data, a priori, eliminates potential information.

Leng et al. (2002) states that “cointegration tests are valid for only stable nonstationary processes.” This assertion places an unnecessary pre-test restriction on time series data. A long-run equilibrium relationship may be subject to many, many influences that cause the relationship to take different forms. Cointegration simply attempts to determine whether a set of variables exhibit the tendency, over the long run, to return to an attracting space. For instance, Leng (2006b) concludes that a structural break in the combined ratio occurred (approximately) in 1981. She further concludes that the combined ratio followed an AR(2) process only during the pre-break period. This indicates a change in dynamics—but the change could easily be limited to short-term dynamics. This result does not eliminate the possibility of the underwriting margin and interest rates having a long-run relationship.

The objective of the univariate pre-testing portion of a cointegration analysis is only to discuss the significance of finding whether the respective variables are stationary in mean, or, to use Granger’s term, persistent.
While I refer to unit root tests, I am not testing for a specific form of nonstationarity.\textsuperscript{11}

**Pre-Testing and Time Trends**

It is my opinion that the inclusion of a time trend factor in a unit root test needs to be given very careful consideration. Harrington and Yu (2003) emphasize the comments of Dejong et al. (1992), who state that the trend “need not literally be part of the data generation process, but may be viewed as a substitute for a complicated and unknown function of population, capital accumulation, technical progress, etc.” There are pitfalls, though, in using a generic trend factor as a substitute for a “complicated and unknown” functional influence. The foremost problem is the removal of information that can be measured and assessed to a known and very legitimate influential factor. In the case of the property-liability underwriting margin, the interest rate is widely accepted as a factor in insurance pricing.

Harrington and Yu (2003) identify other factors affecting underwriting margins over the time period of their study—breakdown of bureau rating systems and growth of direct writers—but the lumping together of these factors (with the possible effects of the interest rate) into a trend removes the ability of a researcher to analyze the particular component effects. A major motivation, in my view, of the development of cointegration analysis was to give researchers the ability to distinguish between generic trend movements (“everything is going up”) and specific estimators regarding the long-run and the short-run relationship between variables. If factors such as the breakdown of bureau rating systems and the growth of direct writers could feasibly and reliably be measured in a time series data set, then these variables would possibly be the subject of a cointegration analysis involving the underwriting margin.

Few economic processes can be directly attributed to the mere passing of time. Dejong et al. (1992) repeatedly refer to the plausibility of using a trend factor when analyzing a time series. To this end, I think it is important for trend factors to be subject to some of the same scrutiny as any other “explanatory” variable.\textsuperscript{12} For instance, a positive time trend factor for underwriting margins makes little sense in the very long run because underwriting margins cannot increase forever. A quadratic time trend may fit a particular time period (as Harrington and Yu show), but a quadratic trend allows for only one bend-point in the trend (see Figure 1). In the limit, neither of these underwriting margin specifications can exist, so any use of these types of trend factors should be regarded as temporary. It can certainly be argued that the mere finding of a statistically significant trend factor is an indicator of the presence of nonstationarity in mean.\textsuperscript{13} The very
long run notwithstanding, it should be acknowledged that all estimates are most applicable to the data and time period being evaluated.

Estimating Cointegration Models

The cointegration technique comprises two sets of estimators—the cointegrating equation and the error-correction model.\textsuperscript{14} The cointegrating equation measures the long-run equilibrium relationship between two (or more) nonstationary variables, while the residuals of this equation, if stationary, contain information about the variables’ short-run dynamics. A complete equilibrium relationship exists only if these (stationary) error terms possess a statistically significant relationship with the (stationary) first differences of at least one of the cointegrating variables. Finding this result is necessary because the residuals of the cointegrating equation represent temporal deviations from the long-run equilibrium attractor, and it is expected that at least one of the cointegrating variables will consistently adjust towards the attractor. Haley (1993) showed that the underwriting margin (insurance prices) makes such an adjustment.\textsuperscript{15} If a statistically significant error-correction is not found in the second step of the estimation, spurious correlation is a likely conclusion.

Cointegration Testing Results

A two-step cointegration analysis was performed on the 90-day Treasury bill rate for new issues and the aggregate underwriting margin for stock property-liability companies, 1930–2000. The estimated cointegrating equation is

\[
ru = 3.055 - 1.197rf
\]  

where \(ru\) = underwriting margin and \(rf\) = 90-day T-bill rate. Both estimated parameters are significant at the 1% level, the adjusted r-square is .376, and the F-ratio is 43.16. These results compare quite favorably to Haley (1993), whose time period was 1930–1989. This equation is representative of the long-run equilibrium relationship between the underwriting margin and the risk free interest rate. The deviations from this equation (potentially) represent the short-run dynamic activity surrounding the equilibrium relationship between the aggregate underwriting and the 90-day T-bill rate.

The second step of the two-step analysis is concerned with verifying the short-run equilibrium adjustments using two error-correction models (ECM). These models take the form
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where \( g = 3 \) and was chosen by minimizing the AIC. The estimated results of the ECM are reported in Table 1. The ECM results show a strong negative relationship (–0.464) between the short-run deviations from the cointegrating line and changes in the underwriting margin. The negative sign on this ECM coefficient conforms to theory by showing that prices tend to increase when the actual underwriting margin is less than the value predicted by the equilibrium cointegrating equation.

The estimated error terms from equation (3) are presented in Figure 2. The error terms appear to be stationary, as they make a consistent effort to return to equilibrium. Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests of these error terms reject the null hypothesis of a unit root, also indicating stationarity.\(^{16}\)

### Table 1. Estimated Parameters of Error-Correction Models

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>( \Delta r_u )</th>
<th>Coefficient</th>
<th>( t )-value</th>
<th>( \Delta r_f )</th>
<th>Coefficient</th>
<th>( t )-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>–0.164</td>
<td>–0.43</td>
<td>0.093</td>
<td>0.71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( z_{t-1} )</td>
<td>–0.464</td>
<td>–3.63*</td>
<td>0.049</td>
<td>1.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta r_{f,t-1} )</td>
<td>0.281</td>
<td>0.66</td>
<td>0.0800</td>
<td>0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta r_{f,t-2} )</td>
<td>0.900</td>
<td>2.38</td>
<td>–0.346</td>
<td>–2.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta r_{f,t-3} )</td>
<td>–0.080</td>
<td>–0.19</td>
<td>–0.088</td>
<td>–0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta r_{u,t-1} )</td>
<td>0.254</td>
<td>2.08</td>
<td>0.026</td>
<td>0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta r_{u,t-2} )</td>
<td>0.024</td>
<td>0.20</td>
<td>0.036</td>
<td>0.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta r_{u,t-3} )</td>
<td>–0.260</td>
<td>–2.28</td>
<td>0.065</td>
<td>1.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( z = \) cointegrating equation error terms, \( \Delta r_f = \) changes in the risk-free rate, \( \Delta r_u = \) changes in underwriting margin

* significant at the 1% level. The 1% standard \( t \)-distribution critical value for \( N = 60 \) is \( \pm 2.66 \).

\[
\Delta r_{u,t} = \alpha_1 + \rho_1 z_{t-1} + \sum_{i=1}^{g} \Delta r_{f,i-1} + \sum_{i=1}^{g} \Delta r_{u,i-1} + e_1
\]

\[
\Delta r_{f,t} = \alpha_2 + \rho_2 z_{t-1} + \sum_{i=1}^{g} \Delta r_{f,i-1} + \sum_{i=1}^{g} \Delta r_{u,i-1} + e_2
\]
A few features in Figure 2 are worth noting. The mid-1940s show some abnormal fluctuation, and the mid-1980s severe underwriting cycle is evident, as is 1992's Hurricane Andrew. It also appears that, starting sometime in the mid-1980s, the behavior of the error term series \textit{may} have changed somewhat. This could indicate some sort of structural change, but it does \textit{not} take away from the cointegrating conclusion a long-run attractor.\textsuperscript{17}

The possibility that the series of estimated error terms contains a structural break is a topic for further investigation. The series of error terms in Figure 2 represent short-term dynamics and possess many of the same characteristics as the aggregate underwriting margin series in Figure 1. The main difference is that the estimated error terms series has been "flattened" by the removal of equilibrium forces provided by the interest rate.

Figure 3 is a phase space plot of the error terms.\textsuperscript{18} A very reasonable interpretation of the diagram is that a stable attractor exists and the aggregate property-liability underwriting margin has a long-term equilibrium relationship with the risk-free interest rate.

The exact attainment of equilibrium is represented by the origin on the graph. It is clear the $(e_t, e_{t+1})$ pairs have a tendency to move towards the area of the origin. The large deviations from exact equilibrium (mostly in the negative quadrant) are associated with the well-known outlying

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\textbf{Fig. 2.} Estimated error terms from cointerating equation, 1930–2000.
market events or situations already mentioned—mid-1940s regulatory uncertainty, the mid-1980 liability crisis, and 1992’s Hurricane Andrew.

Concluding Comments

It is best if important conclusions are made with minimal assumptions and conditions. Nonstationarity can take a variety of forms (as evidenced by the variety of tests available). Mean nonstationarity of interest rates and the property-liability underwriting margin is a sound conclusion, and the detailed consideration of a single DGP existing for the entire 1930–2000 period is unnecessary (for present purposes). The cointegration analysis presents strong evidence of a long-run equilibrium relationship between the 90-day T-bill rate and the underwriting margin. This conclusion agrees with the theoretical models of insurance pricing and affirms the results of Haley (1993) and Choi, Hardigree, and Thistle (2002).

A cointegration analysis, however, cannot be all-encompassing. The variations in the 90-day T-bill rate do not explain all of the variations in underwriting margin. There are clearly other factors that influence the time series behavior of underwriting margins. Regulatory changes, the evolution of rating bureaus into advisory organizations, and changes in taxation all have affected the insurance industry. Yet the aggregate underwriting margin and the 90-day T-bill rate have maintained a long-run relationship through these changes. The error terms generated by removing the long-run equilibrium influence of interest rates form a new time series and still
contain information regarding the (aggregate) short-term dynamics of the property-liability insurance market.

The cointegration methodology differs philosophically from Leng et al. (2002), Harrington and Yu (2003), Leng (2006a, 2006b), and Leng and Meier (2006). The data contain information that needs to be carefully extracted. By taking the incremental approach of determining nonstationarity in mean of the underwriting margin and the interest rate, I acknowledge a feature of the data without asserting a specific DGP, removing information, or eliminating possible models.

It is not my intention, though, to endorse a wide-open approach to data collection. The intent here is to emphasize that researchers need to be careful when extracting information from data. Too heavy a reliance on univariate analysis ignores the complexities and influences of other variables. Placing too many a priori requirements on a data set eliminates information, and a sort of “paralysis-by-analysis” can occur. The main requirement of time series is continuity of objective—i.e., the same variable being measured in each observation. I believe this to be true for both the 90-day T-bill rate series and the aggregate underwriting margin for stock property-liability insurers.

In regards to time trends, finding a significant time trend actually supports the conclusion of nonstationarity in mean. But to assert that the time series is trend stationary instead of difference stationary is a much stronger statement resulting in greater limitations. The assertion of a time trend (leaving only the stationary deviations from the time trend) removes information from the data without assigning much meaning to it. Comparatively, the cointegrating regression inserts the interest rate as a regressor in place of the generic time trend factor and achieves essentially the same result—stationary residuals. Only these residuals have a statistically significant relationship with the first differences of the interest rate and allow for a much more meaningful interpretation that conforms to the widely accepted insurance pricing models.

The research process shouldn’t be unidirectional. Feedback and reconsideration need to be done from time to time in an attempt to reconcile differing approaches and opinions.

NOTES


2 Haley (1993) reports only on the cointegration of the 90-day T-bill rate and underwriting margins. The cointegration conclusion of Choi, Hardigree, and Thistle (2002) is just part of a much broader analysis.
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3 The equilibrium notion used here is somewhat different from that of a market clearing equilibrium. For a discussion see Chapter 1, Sections 1.1 and 1.2, of Banerjee et al. (1993) and pages 179–182 of Milgate (1987).

4 The notion of a non-fixed mean holding throughout the entire analyzed time frame is one that’s repeated several times in this article. However, at no point am I asserting a univariate model or character form on the time series.

5 If a constant term is added to equation (1) the linear relationship is referred to as an “affine” relationship. This does not affect the cointegration analysis.


7 Leng and Meier (2006) do acknowledge that the post-break sample is small.

8 Hakkio and Rush (1991) conclude that the length of a data set in a cointegration study is of greater importance than the number of observations.

9 I am not attempting to invalidate, or even disagree with, the competitiveness conclusions of Leng et al. (2002). I’m only disagreeing with their comment that cointegration is not appropriate because of pre-testing conditions.

10 The time period analyzed was 1952–1997.

11 Granger (1997) actually describes determining whether a time series process has a unit root or is stationary with a broken straight line trend as “uninteresting.” Because there are so many other possible models, Granger writes that the question of interest is simply whether a series is persistent—as evidenced by the failure to reject a unit root hypothesis test.

12 The quotes are used to indicate the dubious nature of a trend factor’s ability to provide explanatory power.

13 Banerjee et al. (1993) make this very point (page 6).

14 The Engle and Granger (1987) technique has two explicit estimation steps.

15 Ceteris paribus, when insurance prices increase, underwriting margins increase.

16 The data set contained 71 observations before differencing and lag computation. The DF test was significant at the 10% level, whereas the ADF tests with 1 lag and 2 lags were both significant at the 1% level. The N = 50 critical values of Engle and Yoo (1987) were used for the tests. No intercept term was used in the estimations.

17 This result roughly coincides with the Leng and Meier (2006) finding of a 1986 structural break in the loss ratio for the U.S. and with the Leng et al. (2002) result of the relationship between underwriting margins and interest rates changing during the early 1980s.

18 The diagram has $ε_t$ on the horizontal axis and $ε_{t+1}$ on the vertical axis.

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