Optimal Asset Allocation for Interconnected Life Insurers in the Low Interest Rate Environment under Solvency Regulation

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Abstract: I assess how Basel III, Solvency II, and the low interest rate environment will affect the financial connection between the bank and insurance sectors by changing the funding patterns of banks as well as the investment strategies of life insurance companies. Especially for life insurance companies, the current low interest rate environment poses a key risk since declining returns on investments jeopardize the guaranteed return on life insurance contracts, a core component of traditional life insurance contracts in several European countries. I consider a contingent claim framework with a direct financial connection between banks and life insurers via bank bonds. The results indicate that life insurers’ demand for bank bonds increases over the mid-term but ultimately declines in the long-run. Since life insurers are the largest purchasers of bank bonds in Europe, banks could lose one of their main funding sources. In addition, I show that shareholder-value-driven life insurers’ appetite for risk increases when the gap between asset return and liability growth diminishes. To check the robustness of the findings, I calibrate a prolonged low interest rate scenario. The results show that the insurer’s risk appetite is even higher when interest rates remain persistently low. A sensitivity analysis regarding industry-specific regulatory safety levels reveals that contagion between bank and life insurer is driven by the insurers’ demand for bank bonds, which itself depends on the regulatory safety level of banks. [Key words: Basel III, Solvency II, life insurance, interest rate guarantees, asset allocation, contagion, interconnectedness.] JEL classifications: G11, G18, G22, G28

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INTRODUCTION

The global financial crisis continues to shape European life insurers’ business performance. A lingering uncertain economic environment, prolonged low interest rates, and a changing regulatory framework pressure the stability of the industry. Especially, the current low interest rate environment poses a key risk to the life insurance sector since declining returns on investments jeopardize the guaranteed return on life insurance contracts, a core component of traditional life insurance contracts in several European countries (e.g., France, Germany, Italy).

Both the banking and the insurance sectors are of great importance for the European economy. Banks contribute to financing the European economy and hold lending assets of approximately €46trn. Due to the nature of their business model, life insurers generate a large inflow of premiums and exhibit an accumulation of assets backing their long-term liabilities. As a result, European life insurers had an estimated €8.5trn of assets under management in 2012. As a consequence, they are among the largest institutional investors in Europe and also the largest purchasers of bank bonds owning around 11% of European bank debt.

In recent years, interest rates have been reduced to exceptionally low levels. This is partly the result of a slight downward trend in euro-area risk-free asset returns over the past decades, and mostly the consequence of the worsening sovereign debt crisis in 2011. Low interest rates affect life insurance companies on both the asset and the liability side. Under the risk-based solvency framework of Solvency II, falling bond yields reduce the discount rate applied to determine the present value of future life insurance benefits and thus increase the market-consistent value of liabilities. When the duration of liabilities is greater than that of assets, which is typical for large life insurance companies in Europe, the appreciation of the present value due to low interest rates is larger for liabilities than for assets, thus reducing the company’s solvency. At the same time, reinvestment returns decline.

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2See Insurance Europe and Oliver Wyman (2013).
3German insurance companies practice their financing function for the corporate sector, especially indirectly, namely through the financing of banks. The 2013 Financial Stability Review of the German Federal Bank reports that in mid-2013, the biggest German insurance companies held €515 billion worth or 36% of their capital investment with banks. See German Federal Bank (2013, p. 81).
4For example, at the end of 2013, a 10-year euro-area government bond (all issuers whose rating is AAA) paid a nominal interest rate of 2.24%. Source: European Central Bank (www.ecb.europa.eu/stats).
To be able to fulfill guaranteed returns to policyholders, life insurers will have to adjust their asset allocation. However, while market conditions cause life insurers to look beyond sovereign bonds, financial regulators make alternatives less palatable. Consequently, life insurance companies may have difficulty fulfilling their payout promises in the future, thus possibly raising overall systemic risk.\(^5\)

With the attempt to enhance market stability, European regulatory frameworks for the financial industry have been changing significantly. Within the banking sector, regulation was enhanced from Basel I in 1988 to Basel II in 2004 up to the most recent change of Basel III in 2010, which came into power in 2014. Similarly, in 2009, the European Parliament passed the Solvency II Directive, which, among other things, will fundamentally alter the capital requirements for insurance companies in the European Union.

Both accords have had a largely independent development process subject to inevitable piecemeal negotiations with different stakeholders. Basel III creates an international standard for banking regulation and supervision, and aims at providing sufficient capital to absorb losses within each of the three risk categories market risk, credit risk, and operational risk. In addition, regulators establish liquidity requirements that promote long-term funding. As a result, banks should become safer; therefore, the cost of funding could decrease as a consequence of higher capital levels.\(^6\)

Solvency II, on the other hand, is the first attempt to develop a fully risk-based solvency standard for the European insurance industry. Capital requirements will be based on the overall risk situation of an insurance company. In addition, insurance companies are encouraged to match their investments more closely to their liabilities.

Given the scale and importance of insurers’ investments into bank bonds, any shift in their asset allocation caused by economic conditions or reforms of financial regulation could have a distorting effect on the connectedness of the banking and insurance sectors. Thus, my aim is to assess how Basel III and Solvency II in the context of the current low interest rate environment will affect the stability of the connection between the two sectors by changing the funding patterns of banks and the investment strategies of insurance companies. More precisely, I address the following questions:

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\(^5\)During their December 2012 meeting, the European Systemic Risk Board (ESRB) identified the low interest rate environment as a potential risk for financial stability in the EU in the medium term. See ESRB (2012).

\(^6\)Using more equity financing should lower the riskiness of a bank’s equity and hence of its debt. See Slovik and Cournède (2011).
To what extent will life insurers’ demand for investments into banks change under Solvency II, given potential changes in the banking industry due to Basel III?

How does this affect life insurers’ equity requirements under Solvency II?

How does insurers’ asset allocation change in a prolonged low interest rate environment?

Are there combinations of industry-specific regulatory safety levels that promote contagion effects between the two sectors?

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature on financial regulation and the interconnectedness between banks and insurance companies. Section 3 introduces the model framework specifying the characteristics of each industry as well as their connection. In section 4, I describe the data and calibration adopted. Section 5 shows the implications for life insurers’ asset allocation from a shareholder value point of view. Subsequently, I study the insurers’ asset allocation in a low interest rate environment and present my findings on contagion effects for different regulatory safety levels. Section 6 concludes.

LITERATURE REVIEW

Two strands of literature are vital for this study: research on financial regulation and studies on the interconnectedness between banking and insurance. The literature on financial regulation is extensive and mainly concentrates on qualitative analyses and comparisons of potential regulatory effects. The Bank for International Settlements (2011) gives an overview of the Basel Accords, the framework, and implementing measures. In the context of Solvency II, Eling et al. (2007) describe the development and main characteristics.

Gauging the consequences of Solvency II and the Basel Accords, Al-Darwish et al. (2011) study possible unintended effects of both frameworks and ultimately suggest a closer communication among banking and insurance regulators. Another qualitative study comparing the capital standards of Basel II/III and Solvency II is done by Gatzert and Wesker (2012). Among other things, the authors conclude that due to the complexity of both regulatory systems, further research is needed to analyze adverse interaction effects between the two regulatory systems.

A recent paper by Laas and Siegel (2013) critically studies regulatory developments in Europe and their effects for the financial sector. The authors focus on accuracy and regulatory consistency of capital charges implied under Basel III and Solvency II. Based on their calculations, they
report that capital requirements for insurers are often more than twice as high as charges for banks. Considering the customers’ perspective, Schmeiser and Wagner (2013) illustrate numerically how minimum guaranteed interest rates should be set by the regulator in order to maximize policyholders’ utility level.

Analyses regarding the risk sharing between banks and insurance companies under financial regulation can be found, for example, in the work of Allen and Carletti (2006). The authors develop a model of financial intermediation with both banking and insurance sectors and find how risk is shared efficiently between the industries. Furthermore, the authors show how credit risk transfer can lead to contagion between the sectors and thus increases the risk of financial crises. Ultimately, they argue that an idiosyncratic shock to the insurance industry can potentially be propelled back to the banking system, endangering its stability. Expanding this model, Allen and Gale (2007) study how inefficient capital regulation can lead to credit risk transfer as a result of regulatory arbitrage, which, in turn, can increase systemic risk.

Little research has been conducted on empirically measuring the interconnectedness among and the systemic risk of financial institutions. A recent analysis by Chen et al. (2013) constructs a systemic risk measure to examine the interconnectedness between banks and insurers by the use of high-frequency data on credit default swap spreads and intra-day stock prices. The authors find evidence of a significant bi-directional causality between insurers and banks. However, the impact of banks on insurers is stronger and of longer duration than the impact of insurers on banks.

Another recent empirical study, by Slijkerman et al. (2013), finds significant downside dependence by investigating the downside risk of European insurers and banks, since these hold numerous cross-exposures and are heavily exposed to the real economy. The authors conclude that the probability of a crash is lower when European banks diversify across other sectors while it becomes higher when they increase size within the banking sector.

Examining the interconnectedness and systemic risk in the US financial industry with respect to certain insurance-specific events (e.g., natural catastrophes), Rauch et al. (2013) empirically analyze how insurance-specific events influence returns on the entire financial industry. The authors use event study methodology and regression analyses. Their main finding is that there is only a very low degree of inter-sector interconnectedness in the financial sector. Hence, they do not find strong abnormal stock market reactions for insurance companies, banks, and banks with insurance business during times of intra-sector-specific events, indicating
no spillover effects. They summarize that there is no need for tighter regulation.

Despite the existence of empirical studies that analyze the interdependence between banks and insurance companies, theoretical analyses taking into consideration recent regulatory changes are rare. Among these few studies, the authors do not consider the interconnectedness of the two sectors. Hence, present studies do not allow an insight into possible reciprocal effects of both regulatory frameworks. By concentrating on banks and life insurance companies, whose financial connection is empirically well documented, I study the optimal risk policy of life insurance companies by explicitly taking into account Basel III and Solvency II. Thus, this paper closes a major gap in the academic literature.

**MODEL FRAMEWORK**

The model captures the main characteristics of the banking and life insurance industry and focuses on the recent interest rate development and the financial connection between banks and insurers as well as the regulatory changes both sectors face. The model consists of a bank offering corporate loans and an insurer offering participating life insurance contracts.

I develop a stylized model of a bank providing loans that are financed by equity capital, deposits, and additional bank debt (bank bonds). For the insurance company, the starting point is a stylized life insurer with an outstanding stock of participating life insurance contracts including minimum guarantees and an empirically calibrated asset structure. Figure 1 gives an outline of how bank and life insurer are connected.

On the stylized balance sheet from Figure 1, the life insurer’s asset side consists of four asset classes: stocks, corporate bonds, bank bonds, and sovereign bonds. The financial connection between bank and life insurer stems from the insurer’s share of investments held in bank bonds.

To impose financial regulation, I require both companies to comply with the regulatory safety levels of Basel III and Solvency II, respectively. I use stochastic balance sheet projections for the bank and the insurance company and study the insurer’s portfolio choice and resulting demand for bank bonds from a shareholder value point of view.

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7For example, the paper by Laas and Siegel (2013) examines and quantifies capital charges under Solvency II, Basel II, and Basel III for an empirically calibrated asset portfolio of an average (Swiss) life insurance company at a given point in time.

8See for example EIOPA (2013b), Insurance Europe (2013), and German Federal Bank (2013).

9See Section “Solvency and Capital Requirements.”
Following the regulatory one-year risk horizon of Solvency II, I choose to maximize the insurer’s net shareholder value over the same period. Thus, the insurer maximizes its net shareholder value by optimizing the one-year-ahead investment strategy while taking into account the equity capital requirement given by Solvency II.

The following actions take place at the beginning of each period: The bank sets the amount of equity capital and liquidity according to the regulatory minima given by Basel III. The life insurer optimizes portfolio weights from a shareholder value point of view by taking into account the constraints given by Solvency II. At the end of each period, both companies realize asset returns. The bank has to pay interests on deposits and debt. If the bank defaults, losses are realized in the insurer’s bank bond portfolio. Policyholders’ yearly surplus participation depends on the insurer’s respective asset return.

The complex interrelations in the model, e.g., a revolving asset structure, changing minimum guarantees, and profit participations, prevent the derivation of a closed-form solution. Hence, I solve the insurer’s optimization problem using numerical methods.

**Interest Rate Dynamics**

In the model, the term structure of risk-free interest rates serves as the main driver for the return on securities in the market. In order to simulate the term structure of risk-free interest rates, I employ the model proposed by Cox et al. (1985) (CIR model). Hence, the underlying short rate process under a risk-neutral measure $Q$ can be written as

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10This is due to nested max-operators with stochastic variables in the objective function (see Equation (21)).

11See Brigo and Mercurio (2006).
where $W(t)$ is a standard Brownian motion, $r(t)$ is the instantaneous interest rate, and $\kappa$ defines the speed of mean reversion, $\theta$ the long-term mean, and $\sigma_r > 0$ the volatility of the short rate process. In addition, $r(0) = r_0$ represents the initial value of the short rate.

Using the affine term structure representation, zero coupon bond prices at time and for maturity are given by

$$B(t,T) = A(t,T)e^{-H(t,T)r(t)},$$

where $t$ is the time spot and $T$ is the maturity time of the bond. $A(t,T)$ and $H(t,T)$ are given by

$$A(t,T) = \frac{2\kappa\theta}{\sigma_r^2} \cdot \ln \left( \frac{2\gamma e^{(\kappa + \gamma)(T-t)/2}}{(\kappa + \gamma)(e^{\gamma(T-t)}-1) + 2\gamma} \right)$$

and

$$H(t,T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\kappa + \gamma)(e^{\gamma(T-t)}-1) + 2\gamma}$$

with

$$\gamma = \sqrt{\kappa^2 + 2\sigma_r^2}.12$$

Under the subjective probability measure $\mathbb{P}$, the short rate process changes to

$$dr(t) = (\kappa \theta - (\kappa - \lambda \cdot \sigma_r) r(t))dt + \sigma_r \cdot \sqrt{r(t)}dW^{\mathbb{P}}(t).$$

and $\gamma$ changes to

$$\gamma = \sqrt{\kappa^2 + 2\sigma_r^2},$$

where $\lambda$ represents the market price of risk.14

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12See Björk (2009).
13See Brigo and Mercurio (2006).
14The parameter $\lambda$ represents the market price of risk, where negative values indicate a positive risk premium for holding long-term bonds. As thoroughly described in Brigo and Mercurio (2006), $\lambda$ connects the real-world measurement to the risk-neutral measurement.
The Bank

At time $t = 0$, the bank holds a portfolio of loans with initial value $A_0^B$. I assume the return on the loan portfolio to be normally distributed with mean $\mu^A_B$ and standard deviation $\sigma^A_B$, that is, $r_A^B \sim N(\mu^A_B, \sigma^A_B)$. The value of the loan portfolio at time $t$ is then $A_t^B = e^{t \cdot r_A^B} \cdot A_0^B$. The bank’s capital structure is composed of equity capital ($E_t^B$), deposits ($D_t^B$), and a portfolio of issued bank bonds denoted $\tilde{B}_t^B$. Deposits are the most senior claim, and the interest rate on deposits is assumed to equal the one-year risk-free rate as provided by the CIR model.

At time $t = 0$, equity capital fulfills the regulatory capital ratio according to Basel III regulation ($k_t^B$) such that $E_0^B = A_0^B \cdot k_t^B$. At time $t$, the bank’s equity capital satisfies

$$E_t^B = A_t^B - \left( \tau_t^B + D_t^B \right) / L_t^B,$$

where $L_t^B$ represents the bank’s liabilities at time $t$.

Debt

To find the market value of a single bank bond $B_t^B$, I determine the respective cash flow. Therefore, I forecast the development of the bank’s assets and liabilities until the maturity of the bond. If the bank does not default during the term of the bond, the investor collects yearly interest payments and a repayment of the bond’s face value at maturity. However, in the case of bankruptcy, the investor will receive a recovery value but no further interest payments afterwards. Finally, I discount the resulting payouts.

For an issued bond with face value $B^{FV}_t$ and maturity $T^B_t$, the bank has to pay the coupon $B^C_t$. I evaluate the bank’s solvency situation at the end of each year until the maturity of the bond, that is, $t \in \{1, 2, \ldots, T^B_t\}$. The fair coupon $c^*$, which compensates investors for the corresponding default risk, is the unique coupon so that the face value equals the present value at the time of issuance. $c^*$ is then the fair coupon rate at the bonds’ issue date. I denote the nominal coupon as $B^C_t = c^* \cdot B^{FV}_t$. 
For the valuation of the bond at time $t$, the investor calculates the market value of the bond. Therefore, I introduce the following stopping time:

$$
\tau^B = \min\{t \in \{1, 2, \ldots, T-1, T\} | E_t^B < 0\}. 
$$

(9)

$\tau^B$ can be interpreted as the first point in time when the bank defaults. In case of default, the investor directly realizes a loss, i.e., a write-down of the face value. The investor's loss writes out as $\phi \cdot \tilde{B}_t^F$, where

$$
\phi = \begin{cases} 
\min \left\{ 1; \frac{(D_t^B + \tilde{B}_t^F) - A_t^B}{\tilde{B}_t^F} \right\} & \text{if } E_t^B < 0 \\
0 & \text{otherwise},
\end{cases}
$$

(10)

with $\tilde{B}_t^F$ representing the total face value of the bond portfolio issued by the bank prior to the bank's default. In other words, shocks to a bond investor come in the form of a percentage loss ($\phi$) in asset values.\(^{16}\) Since the market value of the bond at time $t$ is equal to the expectation of the sum of the discounted cash flows under a risk-neutral measure $Q$, it can be computed as follows:

$$
MV_{t_0}(B^B) = \mathbb{E}^Q \left[ \min\{T^B + t_0, \tau^B - 1\} \sum_{j = t_0 + 1} \text{interest payments } c^* \cdot B_j^{FV} \cdot e^{-t \cdot r_{t_0}} + \right.
\left. I_{\{\tau^B > T^B + t_0\}} \cdot B_0^{FV} \cdot e^{-(T^B + t_0)rf} \right]
$$

(11)

\(^{15}\)I find $c^*$ by valuing the bonds' discounted expected cash flows and then iterating over $c$ until I find the coupon value $c^*$ such that the face value equals the present value. See Pennacchi (2011).

\(^{16}\)A similar approach is used in Bluhm and Krahnen (2014).
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where the indicator function \( I \{ \tau^B \leq T^B + t_0 \} \) is equal to one if the default time \( \tau^B \) is at or before maturity \( (T^B + t_0) \) and \( r_{f(t_0,j)} \) represents the risk-free interest rate at time \( t_0 \) with maturity \( j \). As a result, the market value of the bond directly depends on the risk situation of the bank.

The Life Insurer

I model a stylized life insurance company with an outstanding stock of life insurance contracts including historical minimum return guarantees. I focus on participating life insurance contracts similar to those offered in many European countries. I assume that the insurer continues to sell a homogeneous savings product with a minimum guaranteed return and fixed time to maturity. Collected premiums will be invested into four asset classes: stocks \((S)\), a portfolio of corporate bonds \((CB)\), a portfolio of bank bonds \((B)\), and a portfolio of government bonds \((GB)\). Neither mortality nor surrender risk is assumed, thus only financial risk is taken into consideration. Benefit payments to policyholders are made only at the time of maturity of each contract. Furthermore, I do not consider transaction costs.

The insurer’s initial assets are denoted by \( A^I_0 \). At time \( t = 0 \), the insurer’s shareholders endow the company with equity capital \( e^I_0 \). Owing to agency costs and acquisition expenses, equity endowment is assumed to imply up-front frictional costs, which are modeled by a proportional charge \( \varphi \geq 0 \).19

17 Note that \( \min \{ \phi \} = \infty \) by convention, so that \( \tau^B = \infty \) if the bank does not default in \( t \in \{ 1,2,\ldots,T \} \).
18 A similar product is used in Kling et al. (2007) and Berdin and Gründl (2014).
19 This is a common approach in the literature; see, for example, Froot (2007).
I denote the resulting equity capital for the company as $E^I_0 = e^I_0 \cdot (1 - \varphi)$. The implication is that, all else equal, an additional dollar of equity capital raises the market value of the firm by less than one dollar. At the beginning of each year, the insurer decides on portfolio weights $w = [w_1, w_2, w_3, w_4]$ for the four asset classes (Table 1).

To capture the dynamics of the insurer’s asset portfolio, such as the adjustment to changing interest rates, I introduce two frictions. First, I utilize a lag in the average return on bond portfolios. I assume the portfolios of bank bonds, corporate bonds, and government bonds to be revolving with an average time to maturity according to Table 1. This implies that each bond portfolio consists of multiple cohorts of bonds with a similar maturity but a different remaining time to maturity (Figure 1). In other words, the return on each bond portfolio is given by the 9-year moving average, where 9 represents the constant average time to maturity of the respective asset class, i.e., $T^{CB}, T^{TB}$, or $T^{GB}$.

The second friction I implement is a limitation on the reallocation of the insurer’s portfolio. The insurer is allowed to shift only a proportion $\delta$ of each asset class at the end of every year. While the first friction determines the average return of all bonds in the portfolio, the second friction limits the change in the weight of each asset class.

To translate this into the model, I deconstruct the return on the bond portfolio into the risk-free rate and a premium. This is consistent to the underlying assumption that the risk-free interest rate serves as main driver for the return on securities in the market. For the risk-free part, I use the

\[ E^I_0 = e^I_0 \cdot (1 - \varphi) \]

\[ w = [w_1, w_2, w_3, w_4] \]

\[ T^{CB}, T^{TB}, T^{GB} \]

**Table 1. Asset Classes of the Insurer**

<table>
<thead>
<tr>
<th>Asset class</th>
<th>Notation</th>
<th>Weight</th>
<th>Average time to maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>$S$</td>
<td>$w_1$</td>
<td>$-$</td>
</tr>
<tr>
<td>Bank bonds</td>
<td>$\tilde{B}$</td>
<td>$w_2$</td>
<td>$T^{TB}$</td>
</tr>
<tr>
<td>Corporate bonds</td>
<td>$\tilde{CB}$</td>
<td>$w_3$</td>
<td>$T^{CB}$</td>
</tr>
<tr>
<td>Government bonds</td>
<td>$\tilde{GB}$</td>
<td>$w_4$</td>
<td>$T^{GB}$</td>
</tr>
</tbody>
</table>

20A similar approach is used in Maurer et al. (2013).
average of the risk-free rate at time $t, r_f(t)$, over the last 9 years as provided by the CIR model. I denote the resulting risk-free averages as $r_f^{GB}(t, T_{GB}), r_f^{B}(t, T_{B})$, and $r_f^{CB}(t, T_{CB})$ (according to the average time to maturity of the respective asset class from Table 1). The same approach is used for the risk premium.

To model the insurer’s asset and liability side, I consider a stochastic development of the stock market and the corporate investment spread. For the insurer’s stock holdings, I assume a normally distributed rate of return $r_S$ with mean $\mu_S$ and standard deviation $\sigma_S$, that is, $r_S \sim N(\mu_S, \sigma_S^2)$. The value of the stock investment at time $t$ is then $S_t = e^{r_S t} S_0$.

The returns on corporate bonds and bank bonds are derived by adding a premium to the risk-free rate. I assume the credit spread on corporate bonds $CS^{CB}$ to be normally distributed with mean $\mu_{CB}$ and standard deviation $\sigma_{CB}$, that is, $CS^{CB} \sim N(\mu_{CB}, \sigma_{CB}^2)$. Similar to the risk-free part, I use the average of the credit spread over the last $T_{CB}$ years. The return on the corporate bond portfolio at time $t$ is then given by

$$
r_{CB}^{CB}(t, T_{CB}) = r_f^{CB}(t, T_{CB}) + CS^{CB}(t, T_{CB}).
$$

For the part invested into bank bonds, the credit spread directly depends on the bank’s solvency situation (see Equation (11)). I derive the credit spread on bank bonds at time $t$ as the difference between the fair credit spread and the risk free rate. It follows

$$
CS^B_t = \tilde{c}_t - r_f(t, T_B).
$$

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21 In other words, $r_f^{GB}(t, T_{GB})$ represents the average of the $T_{GB}$-year risk-free rates over the last $T_{GB}$ years at time $t$.

22 See, for example, Maurer et al. (2013, p. 12).
As before, the average of the risk-free rate as well as the credit spread over the last \( T^B \) years is used. The return on bank bond portfolio at time \( t \) is given by

\[
\tilde{r}_{(t,T^B)}^B = \underbrace{\tilde{r}_{(t,T^B)}^B}_{\text{average risk-free rate}} + \underbrace{\tilde{C}S_{(t,T^B)}^B}_{\text{average corporate bond spread}}.
\]

For government bonds, I assume the rate of return at time \( t \) \( (\tilde{r}_{(t,T^{GB})}^G) \) to equal the average \( T^{GB} \)-year risk-free rate over the last \( T^{GB} \) years.

The insurer’s assets at time \( t \) equal

\[
A_t^I = S_t + MV_t(CB_t^I) + MV_t(\tilde{B}_t^I) + MV_t(\tilde{G}B_t^I),
\]

where \( CB_t^I, \tilde{B}_t^I, \) and \( \tilde{G}B_t^I \) represent the portfolios of corporate bonds, bank bonds, and government bonds at time \( t \), respectively.

In analogy to the bond portfolios on the insurer’s asset side, I consider a revolving portfolio of participating life insurance contracts carrying minimum guaranteed returns committed to policyholders at the inception of their contracts.\(^{23}\) I assume that the portfolio of life insurance contracts has a constant average time to maturity of \( T^L \) years. The average value of the minimum guaranteed return of the overall portfolio at time \( t \) is denoted \( \tilde{r}_{(t,T^{GB})}^G \). The development of the guaranteed rate of return of new insurance contracts is based on the development of the 10-year risk-free rate provided by the CIR model.

Following the German Insurance Supervision Code, the reference rate \( (r_{ref}^G) \) for determining the maximum guaranteed interest rate \( r^G \) is equal to 60% of the 10-year moving average of the 10-year German government bond yield. For the guaranteed rate of return of new incoming cohorts of contracts follows

\(^{23}\)The liability portfolio thus consists of several cohorts of contracts with a similar maturity but a different remaining time to maturity.
Consistent with the observed changes in the technical interest rate in recent years, I assume $\omega = 0.5\%$. To reproduce empirically observed dynamics, such as the evolution of the average guaranteed interest rate over time, I adjust the overall portfolio guarantee as

$$
\tilde{r}_t^G = r_t^G \cdot \beta + r_{t+1}^G \cdot (1 - \beta), \tag{17}
$$

with $\beta \in [0, 1]$.

For $t = 1, \ldots, T$, the liability growth rate is given by

$$
r_t^L = \max\left(\tilde{r}_t^G; \alpha \cdot \left(\frac{A_t^I}{A_{t-1}^I} - 1\right)\right), \tag{18}
$$

with $\alpha$ being the participation parameter of the annual return on the insurer’s investment portfolio. The evolution of the insurer’s liabilities, i.e., the aggregated policyholder account in book values, is expressed by

$$
\tilde{L}_t = \tilde{L}_{t-1} \cdot e^{r_t^L}, \tag{19}
$$

where $\tilde{L}_{t-1}$ is the book value of the aggregated account at time $t - 1$ and $r_t^L$ is the liability growth between $t - 1$ and $t$. For regulatory purposes, i.e., to determine the value of the liabilities in the solvency balance sheet, I calculate the market consistent value of liabilities at time $t$ as follows

$$
MCV_t(\tilde{L}_t) = \frac{\tilde{L}_t \cdot \exp \left(\tilde{r}_t^G \cdot T_t^L\right)}{\exp \left(r_{(t,t+T_t^L)}^G \cdot T_t^L\right)}, \tag{20}
$$

where $\tilde{L}_t$ is the book value of the aggregated account at time $t$ and $\tilde{r}_t^G$ is the average guaranteed return to policyholders over the following $T_t^L$.

\footnote{Solvency II postulates the principal of the best estimate for the market consistent valuation of insurance liabilities. Hence I discount the minimum final payment the insurer has to make at the end of the contracts.}
years. Furthermore, $T^L$ represents the average time to maturity of all insurance contracts in the insurer’s liability portfolio and $r_{f_{(t,T^L)}}$ is the $T^L$-year risk-free rate at time $t$ provided by the CIR model. The numerator reflects the year-by-year accrual of the policyholders accounts as it is a common product feature for life insurance contracts.

From $t = 1, \ldots, T$, the insurance company aims to maximize its one-year net shareholder value. The maximization function is

$$SHV_t^I = \max_w \left\{ \exp(-r_{f_{(t,1)}}) \cdot \mathbb{E}^P \left[ \max\{A^I_{t+1} - MCV_{t+1}^I(L_{t+1});0\} - E^I_t \right] \right\}$$ (21)

subject to $\sum_{i=1}^4 w_i = 1$, \hspace{1cm} (22)

$w_i \geq 0$ \hspace{1cm} (23)

and $P(A^I_{t+1} < MCV_{t+1}^I(L_{t+1})) \leq \varepsilon^I_t$, \hspace{1cm} (24)

where $A^I_{t+1}$ represents the stochastic market value of assets at time $t + 1$, $MCV_{t+1}^I(L_{t+1})$ the stochastic market consistent value of liabilities at time $t + 1$, and $r_{f_{(t,1)}}$ the one-year risk-free rate at time $t$. In addition, $\varepsilon^I_t$ is the maximum one-year default probability allowed under Solvency II.

The maximization will be subject to three types of constraints: a budget constraint (Equation (22)) that requires the insurer to invest all of the available capital, a short sale constraint (Inequality (23)), and a solvency constraint that ensures the insurer maintains the regulatory safety level (Inequality (24)).

In this setting, the insurer maximizes the net shareholder value by optimizing the one-year-ahead investment strategy. Due to the solvency constraint, a riskier firm policy, e.g., a more risky asset allocation, will result in higher equity capital requirements. This is because a riskier firm policy results in higher weights assigned to risky assets, which causes a more volatile asset portfolio. Therefore, the probability of liabilities exceeding assets increases. To ensure a survival probability of 99.5%, a larger amount of equity capital is needed. A trade-off situation emerges.

To measure the contagion risk between the bank and the life insurance company in the model, I distinguish between two types of default of the insurer. First, a general default, and second, a bank triggered default which describes a situation in which the insurer defaults due to a bank default
(through Equation (10)). In other words, I track insurer’s defaults that would not have happened if the bank had survived. Therefore, I introduce three events:

A → the insurer defaults at time t \( (A_t^I < MCV_t(\tilde{L}_t)) \),

B → the bank defaults at time t \( (A_t^B < L_t^B) \),

C → the insurer would not default at time t if the bank had not been bankrupt at time t.

In the model, event C can be determined by evaluating the insurer’s equity capital without taking into consideration its loss on the bank bond portfolio. If the resulting equity capital turns out to be positive, the insurer would have survived if not for the bank’s default. For the conditional probability of contagion \( (\pi_{c_t}^I) \) follows

\[
\pi_{c_t}^I = P(A|B,C) = \frac{P(A \cap B \cap C)}{P(B \cap C)}. \tag{25}
\]

\( (\pi_{c_t}^I) \) can be interpreted as the probability that a bank default leads to an insurance default. I use \( (\pi_{c_t}^I) \) to measure the degree of contagion risk in the model.

**Solvency and Capital Requirements**

Over the past decade, regulatory frameworks in the financial services industry in the European Union have changed significantly. The declared goals of regulators are increasing transparency and stability in the financial system. Within the banking sector, regulation has been enhanced from Basel I in 1988 to Basel II in 2004 up to the most recent change of Basel III agreed upon by the members of the Basel Committee on Banking Supervision in 2010. Similarly, over the past decade, insurance regulators have developed a new risk-based solvency framework, Solvency II, that is expected to come into force in 2016.

**Basel III**

Basel III introduces a schedule to increase minimum capital requirements over the next years. In addition, regulators establish liquidity requirements that penalize excessive reliance on short-term, inter-bank funding to support long-term funding. To impose Basel III in the model, in each year, the regulator requires the bank to fulfill the minimum equity capital ratio \( (k^B_t) \) as well as liquidity requirements defined by the Basel Committee (Table 2).25
To improve banks’ ability to absorb shocks arising from financial and economic stress, the liquidity coverage ratio (LCR) ensures an adequate stock of high-quality liquid assets that could easily be sold to meet banks’ liquidity needs in times of crisis. The LCR is specified as

$$LCR = \frac{\text{Stock of unencumbered high-quality assets}}{\text{Total net cash outflows over specified period}} \geq 100\%. \quad (26)$$

This requires a bank’s stock of unencumbered high-quality liquid assets to be larger than projected net cash outflows over a specified time horizon under a stress scenario specified by supervisors. Within the model, high-liquid assets are represented by one-year government bonds. I calculate the denominator of Equation (26) by multiplying the size of liabilities with the rates at which they are expected to run off or be drawn down in a stress scenario, which includes a partial loss of deposits and a significant loss of funding.\(^{26}\)

The second regulatory ratio of the new Basel accord is the net stable funding ratio (NSFR), which addresses capital surcharges proportional to the size of the maturity mismatch between a banks’ assets and liabilities. It is defined as

$$NSFR = \frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} > 100\%. \quad (27)$$

\(^{25}\)See Bank for International Settlements (2011, Annex 4). In contrast to Solvency II, the amount of risk capital for banks under Basel III is not based on a predefined solvency probability at the company level (no explicit probabilistic basis to define requirements).

The NSFR requires banks to hold the ratio of “stable funding” (i.e., equity capital, customer deposits, other long-term sources of funding) to “non-liquid assets” above the predefined regulatory minimum. To derive the NSFR in the model, I weight the bank’s assets and liabilities using the factors proposed by BIS (Bank for International Settlements, 2013).27

Solvency II

The introduction of Solvency II will fundamentally change the capital requirements for insurance companies. The standard aims to reflect the full range of risks faced by insurers on both their asset and liability side to achieve a one-year company solvency probability of at least 99.5%.28 This translates into an upper bound of the one-year company default probability of $\varepsilon^I = 0.5\%$.

To assess capital requirements, I develop an internal model as a parsimonious asset-liability approach. Consistent with Solvency II, capital requirements are calculated based on a 0.5% Value-at-Risk over a one-year horizon.

To calculate the Solvency Capital Requirement (SCR), I proceed in two steps29:

1. Project the insurer’s assets and liabilities over a one-year horizon in order to evaluate the net asset value at time $t + 1$.
2. Discount the value of the 99.5% quantile to time to quantify the amount of equity capital which, invested at time $t$, will enable the insurer to avoid bankruptcy in 99.5% of cases.

Insolvency of the insurer occurs if the insurer’s asset values are lower than the market consistent value of liabilities at time $t + 1$, that is

$$
\frac{ST_{t+1} + MV_{t+1}(\tilde{C}B_{t+1}) + MV_{t+1}(\tilde{B}l_{t+1}) + MV_{t+1}(\tilde{G}B_{t+1})}{A^I_{t+1}}
\quad MCV_{t+1}(\tilde{L}_{t+1}) <
$$

To determine the value of the insurer’s assets at time $t + 1$, note that

$$
A^I_{t+1} = A^I_t \cdot \exp\left(\frac{r_{A^I_t}}{A^I_t}\right),
$$

28See EIOPA (2013c, p. 9).
29See Habart-Corlosquet et al. (2013, p. 9).
where $A_t^l$ is the stochastic market value of assets at time $t + 1$, $A_t^d$ is the deterministic market value of assets at time $t$, and $r_{A_t^s}$ is the stochastic return on assets between time $t$ and $t + 1$. The asset portfolio return $r_{A_t^p}$ results from the weighted average of the individual asset returns given by

$$r_{A_t^p} = \sum_{i=1}^{4} w_i^s \cdot r_{i,t}^s \text{ with } i \in \{ST, CB, B, GB\}.$$  \hfill (30)

For the stochastic aggregated policyholder account at time $t + 1$ follows

$$\tilde{L}_t + 1 = \tilde{L}_t \cdot \exp \left( \max \left( \tilde{r}_{t+1}^G; \alpha \cdot r_{A_t^p} \right) \right),$$  \hfill (31)

where $\tilde{L}_t$ is the deterministic value of liabilities at time $t$, $r_{A_t^p}$ is the stochastic return on assets between time $t$ and $t + 1$, $\tilde{r}_{t+1}^G$ is the average guaranteed return of all insurance contracts in the insurer’s liability portfolio between time $t$ and $t + 1$, and $\alpha$ is the participation parameter of the annual return on the insurer’s investment portfolio. The market consistent value of liabilities at time $t + 1$ is calculated using Equation (20).

Based on the projections of assets and liabilities, a distribution for the stochastic market value of the insurer’s equity capital at time $t + 1$ can be derived from

$$\tilde{E}_t + 1 = ST_{t+1} + MV_{t+1} \tilde{CB}_{t+1} +$$

$$MV_{t+1} \tilde{B}_{t+1} + MV_{t+1} \tilde{GB}_{t+1} - MCV_{t+1} \tilde{L}_{t+1}. \hfill (32)$$

I compute the SCR at time $t$ as the discounted 99.5%-Value-at-Risk of the distribution of $E_t^l$. It follows

$$SCR_t = e^{-\phi( - q_{E_t^l(0.005)})^+}, \hfill (33)$$

with $\phi$ representing the one-period discount rate\textsuperscript{30} and $q_{E_t^l(0.005)}(\varepsilon)$ the value of the $\varepsilon$-quantile of the distribution of $E_t^l$.\textsuperscript{31} This ensures that Inequality (24) holds.

\textsuperscript{30}I use the insurer’s average asset return between time $t$ and $t + 1$.

\textsuperscript{31}See EIOPA (2013c, p. 116).
Notwithstanding its parsimony, the internal model captures interest rate risk, equity risk, and credit risk. The impact of each asset class on the capital requirements is driven by its underlying characteristics.

**CALIBRATION**

In Table 3, I report the empirically calibrated parameters used in the numerical analysis. For the CIR Model, I follow Brigo et al. (2009) and calibrate the parameters $\kappa$, $\theta$, and $\sigma_r$ based on the overnight inter-banking interest rate in Germany. The market price of risk is set to $\lambda = -0.1$. The starting point of the short rate ($r_0$) is based on the December 2013 EONIA value. The development of the bank’s assets is calibrated on the return index of a corporate bond portfolio (Bank of America Merrill Lynch EMU Corporate Bond Index).

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32Until the year 1999, the overnight rate FIBOR was used, and subsequently EONIA. Monthly data from 1983–2013. Source: DataStream.


As for the insurer, the frictional costs on raising equity capital are set to \( \varphi = 0.1 \). The starting values for the average returns on corporate bonds, bank bonds, and government bonds are based on the historical term structure of risk-free interest rates as well as empirically observed risk premia. I calibrate the stock price development on the DAX-Index.\(^{35}\) For the calibration of the risk premium on corporate bonds, I use historical data of the annualized agreed rate of 10-year loans to non-financial corporations as well as

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historical data on the 10-year yield of AAA euro-area government bonds.\(^ {36}\)
I correlate the four stochastic processes based on empirical data (Table 5).

According to the German Insurance Association (GDV), in 2012, on average 20\% of the portfolio was reinvested. Thus, \(\delta\) is set to 0.2. The liabilities’ time to maturity is based on the results of the second Quantitative Impact Study (QIS2).\(^ {37}\) A market survey by Assekurata Cologne (2012) reports an average guaranteed interest rate of 3.19\% for 2012. I use this as a starting point. For the calibration of \(\beta\), I follow Berdin and Gründl (2014) and assume a portfolio of 25 cohorts of contracts with different terms to maturity. Thus, \(\beta\) is set to 0.04. Following the German “Minimum Allocation Decree,” the policyholders’ profit participation is set to 90\% of the insurer’s asset return.\(^ {38}\)

The initial weights of the life insurer’s asset portfolio are set to: \(w_1 = 0.05, w_2 = 0.11, w_3 = 0.20,\) and \(w_4 = 0.64.\)\(^ {39}\
I normalize the starting value of the aggregated policyholder account to 1.


\(^{37}\)More recent impact studies do not report updated figures.

\(^{38}\)According to §4, paragraph 3 of the German “Minimum Allocation Decree” (Mindestzuführungs-verordnung).

\(^{39}\)The figures are based on the 2011 averages of the main asset classes for life insurance enterprises in France, Germany, and Italy. See EIOPA (2013a).
Table 6. Insurer’s Net Shareholder Value Maximizing Portfolio Weights

<table>
<thead>
<tr>
<th>Asset class / Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>0.050</td>
<td>0.040</td>
<td>0.039</td>
<td>0.037</td>
<td>0.033</td>
<td>0.029</td>
<td>0.026</td>
<td>0.022</td>
<td>0.019</td>
<td>0.016</td>
<td>0.015</td>
</tr>
<tr>
<td>Bank bonds</td>
<td>0.110</td>
<td>0.288</td>
<td>0.301</td>
<td>0.286</td>
<td>0.261</td>
<td>0.231</td>
<td>0.201</td>
<td>0.174</td>
<td>0.150</td>
<td>0.129</td>
<td>0.113</td>
</tr>
<tr>
<td>Corporate bonds</td>
<td>0.200</td>
<td>0.160</td>
<td>0.150</td>
<td>0.145</td>
<td>0.149</td>
<td>0.158</td>
<td>0.170</td>
<td>0.181</td>
<td>0.192</td>
<td>0.207</td>
<td>0.223</td>
</tr>
<tr>
<td>Government bonds</td>
<td>0.640</td>
<td>0.512</td>
<td>0.510</td>
<td>0.532</td>
<td>0.557</td>
<td>0.581</td>
<td>0.604</td>
<td>0.624</td>
<td>0.639</td>
<td>0.647</td>
<td>0.649</td>
</tr>
</tbody>
</table>

Table 7. Insurer’s Return on Assets and Average Guaranteed Return

<table>
<thead>
<tr>
<th>Return / Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{A}^{I} )</td>
<td>0.0344</td>
<td>0.0314</td>
<td>0.0303</td>
<td>0.0306</td>
<td>0.0311</td>
<td>0.0321</td>
<td>0.0331</td>
<td>0.0345</td>
<td>0.0355</td>
<td>0.0369</td>
</tr>
<tr>
<td>( r^{G} )</td>
<td>0.0319</td>
<td>0.0313</td>
<td>0.0306</td>
<td>0.0299</td>
<td>0.0293</td>
<td>0.0285</td>
<td>0.0278</td>
<td>0.0272</td>
<td>0.0265</td>
<td>0.0260</td>
</tr>
<tr>
<td>( (r_{A}^{I} - r^{G}) )</td>
<td>0.0025</td>
<td>0.0001</td>
<td>–0.0003</td>
<td>0.0007</td>
<td>0.0018</td>
<td>0.0036</td>
<td>0.0053</td>
<td>0.0073</td>
<td>0.0090</td>
<td>0.0109</td>
</tr>
</tbody>
</table>

Table 8. Insurer’s Net Shareholder Value Maximizing Portfolio Weights for \( \theta = 0.02 \).

<table>
<thead>
<tr>
<th>Asset class / Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>0.050</td>
<td>0.040</td>
<td>0.039</td>
<td>0.037</td>
<td>0.035</td>
<td>0.032</td>
<td>0.028</td>
<td>0.025</td>
<td>0.021</td>
<td>0.019</td>
<td>0.017</td>
</tr>
<tr>
<td>Bank bonds</td>
<td>0.110</td>
<td>0.288</td>
<td>0.303</td>
<td>0.286</td>
<td>0.260</td>
<td>0.228</td>
<td>0.197</td>
<td>0.168</td>
<td>0.143</td>
<td>0.122</td>
<td>0.104</td>
</tr>
<tr>
<td>Corporate bonds</td>
<td>0.200</td>
<td>0.160</td>
<td>0.185</td>
<td>0.215</td>
<td>0.246</td>
<td>0.278</td>
<td>0.307</td>
<td>0.333</td>
<td>0.353</td>
<td>0.368</td>
<td>0.378</td>
</tr>
<tr>
<td>Government bonds</td>
<td>0.640</td>
<td>0.512</td>
<td>0.473</td>
<td>0.462</td>
<td>0.460</td>
<td>0.462</td>
<td>0.468</td>
<td>0.474</td>
<td>0.482</td>
<td>0.492</td>
<td>0.502</td>
</tr>
</tbody>
</table>
I project the bank’s and insurer’s balance sheets 10 years forward under 10,000 CIR and GBM iterations (i.e., the underlying capital market simulation). At the end of every year, I project 10,000 one-year developments of the bank as well as the insurer’s asset and liability portfolio in order to assess the insurer’s portfolio choice and solvency capital requirement.

**RESULTS**

In this section, I apply the model framework and analyze life insurers’ optimal risk policy. Key elements of the analysis are the development of the insurer’s asset allocation, the interaction between asset return and liability growth and the resulting capital requirements according to Solvency II. In a second step, I study how the optimal risk policy is affected by a change in the long-term interest rate level. Subsequently, I analyze how different regulatory safety levels influence the insurer’s asset allocation and the contagion risk between the banking and insurance sectors.

**Optimal Risk Policy**

Figure 3 depicts the shareholder value optimal portfolio composition from \( t = 1, \ldots, 10 \) obtained from the optimization. Since the starting value of the short rate is quite small \( (r_0 = 0.0045) \), the interest rates generated by
the CIR model are also very small within the first years. However, in the long run, the short rate converges to its long-term equilibrium ($\theta = 0.03$), which implies increasing interest rates. Hence, I observe a significant change in the life insurer’s asset allocation.

Given the low interest rates in the first years, the pressure to fulfill interest rate commitments increases and the insurer shifts reinvestments towards more risky asset classes, especially bank bonds (Table 6). In the literature, this behavior is called gamble for redemption or search for yield.\footnote{See, for example, Antolin et al. (2011) and EIOPA (2013b).} As a result, the company’s government bond share reaches its minimum at the end of the second year. At the same time, I observe a decline in stock investments. This is in line with empirical observations over the past years and results from the high capital requirements for stocks under Solvency II.

With interest rates rising in the long run, I observe an increase in the overall time to maturity of the insurer’s assets arising from a shift in reinvestments towards government bonds. While the demand for bank bonds increases in the short term, life insurers are less inclined to invest into bank debt in the long run. This results from the increasing minimum capital and liquidity requirements for banks during the considered time period (see Table 2), generally decreasing banks’ default risk as well as the return on bank bonds.

The second dynamic I examine is the interaction between the return on assets and the actual growth rate of the insurer’s liabilities, i.e., the policyholder account (Figure 4). Due to the revolving investment strategy and the lower average time to maturity of the asset side, the return on life insurer’s assets adjusts quicker to the low interest rate environment than the growth rate of the liabilities, which includes high guarantee commitments to policyholders.\footnote{Recall that due to the revolving investment strategy, older bonds with relatively high coupons mature and are replaced by new bonds with lower coupons.} Although interest rates remain low and the reference interest rate adjusts downward, the average growth rate of the liabilities decreases slowly due to the large proportion of expensive insurance contracts in the liability portfolio. This poses a serious threat to the company’s overall profitability and solvability over the mid-term (Table 7).

Thus, in a low interest rate environment from a shareholders’ point of view, the smaller the gap between the average guarantee of the outstanding liability structure and the asset return (lowest return on assets in year 3, Table 7), the higher the incentive to invest into risky assets (highest demand for bank bonds in year 3, Table 6). Despite the observed change in the asset allocation, the insurer’s asset return falls below the average guaranteed interest rate in year 3.
Fig. 4. Life insurer’s return on assets and return on liabilities.

Fig. 5. Life insurer’s required equity capital to maintain a 99.5% survival probability.
When the overall asset return is very close to or even smaller than the average guaranteed yield of the liability portfolio, the net shareholder value becomes negative since policyholder guarantees are paid using equity capital.

In Figure 5, I report the life insurer’s capital requirements to comply with the regulatory default probability of 0.5% as given by Solvency II. Although the asset return declines slowly, the required equity capital increases quickly. The drastic increase in capital requirements can be explained by two effects. First, after a period of declining interest rates, the average guaranteed interest rate starts to exceed the yields on high-rated government bonds (see year 3 in Figure 4). This increases the company risk and thus capital requirements. Second, the declining asset returns generally pressure the company to increase its appetite for risk.

The substitution of government bonds with higher-yielding, more risky investments widens the duration gap, increases the volatility of the asset portfolio, and thus leads to rising capital charges. The required equity capital peaks in year 6.\textsuperscript{42} Since in the model the insurer is able to shift only a part of its portfolio every year, capital charges decline slowly.

\textsuperscript{42}Note that the average required equity capital ratio from \( t = 1, \ldots, 10 \) amounts to 11.14\%.
### Table 9. Insurer’s Return on Assets and Average Guaranteed Return for $\theta = 0.02$

<table>
<thead>
<tr>
<th>Return / Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{A^I}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0.0339</td>
<td>0.0304</td>
<td>0.0285</td>
<td>0.0283</td>
<td>0.0285</td>
<td>0.0294</td>
<td>0.0302</td>
<td>0.0313</td>
<td>0.0321</td>
<td>0.0329</td>
<td></td>
</tr>
<tr>
<td>$r^G$</td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0319</td>
<td>0.0313</td>
<td>0.0306</td>
<td>0.0299</td>
<td>0.0291</td>
<td>0.0283</td>
<td>0.0278</td>
<td>0.0269</td>
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<td></td>
</tr>
<tr>
<td>$(r_{A^I} - r^G)$</td>
<td>0.0020</td>
<td>-0.0009</td>
<td>-0.0021</td>
<td>-0.0016</td>
<td>-0.0006</td>
<td>0.0011</td>
<td>0.0024</td>
<td>0.0044</td>
<td>0.0059</td>
<td>0.0074</td>
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### Table 10. Insurer’s Equity Capital Requirements under Different Interest Rate Calibrations

<table>
<thead>
<tr>
<th>SCR / Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0.03$</td>
<td>0.0182</td>
<td>0.0914</td>
<td>0.1138</td>
<td>0.1286</td>
<td>0.1341</td>
<td>0.1342</td>
<td>0.1315</td>
<td>0.1265</td>
<td>0.1205</td>
<td>0.1157</td>
</tr>
<tr>
<td>$\theta = 0.02$</td>
<td>0.0747</td>
<td>0.1052</td>
<td>0.1271</td>
<td>0.1414</td>
<td>0.1476</td>
<td>0.1477</td>
<td>0.1415</td>
<td>0.1338</td>
<td>0.1273</td>
<td>0.1217</td>
</tr>
</tbody>
</table>
As the internal model is based on an asset-liability approach, the observed capital requirements are attributable to a wide duration gap, which is caused by the decreasing shares of government bonds in the asset portfolio. When the share of risk-free investments increases, the duration gap narrows and the capital requirements start to decline.

**Interest Rate Sensitivity**

To account for a different interest rate scenario and in order to check the robustness of the findings, I calibrate a different capital market setting. By changing the long-term equilibrium interest rate \( \theta \) in the CIR calibration, I am able to reproduce a structurally different interest rate level, as the CIR model features a mean reverting behavior. Whereas the baseline calibration reproduces a scenario where interest rates gradually recover towards a higher level, I now reduce the long-term equilibrium interest rate from \( \theta = 0.03 \) to \( \theta = 0.02 \) to generate a situation in which interest rates stay at a similar level as observed in Germany in 2013.43

Figure 6 illustrates the results for the insurer’s portfolio weights given a lower interest rate environment. Compared to the baseline calibration, the insurer shifts even more assets towards risky investments. Therefore, the insurer’s government bond share decreases (Table 8). However, the demand for corporate bonds is higher than for bank bonds. At the same time, the proportion of stocks slightly increases. As before, the demand for bank bonds decreases in the long run due to the increasing minimum capital and liquidity requirements for banks.

A change in the long-term equilibrium interest rate also has an effect on the insurer’s return on assets and liability growth. As a result of lower interest rates, the growth rate of liabilities decreases since the guaranteed rate of return of new contracts is smaller than in the baseline calibration (Table 9). However, the insurer’s return on assets also decreases. Figure 7 depicts the results for both figures. I observe that the insurer’s asset return falls below the average guaranteed interest rate for a period of about four years. Although the insurer’s asset return exceeds the average guaranteed yield in the long run, the return on assets is smaller than in the baseline calibration.

As the low interest rate environment pressures the life insurer to shift reinvestments into risky asset classes, capital requirements to comply with the regulatory default probability of 0.5% increase faster (Figure 8). Since the insurer’s asset return is insufficient to keep up with the increase in liabilities for several years, I observe an upward shift in capital charges. On

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43A similar approach is used in Reuss et al. (2013, p. 15).
Fig. 7. Life insurer’s return on assets and return on liabilities for $\theta = 0.02$.

Fig. 8. Solvency capital requirement under different interest rate environments.
average, the amount of capital increases by around 14% compared to the baseline calibration.\footnote{Note that the average required equity capital ratio from } Despite the change in the interest level, the required equity capital again peaks in year 6 (Table 10).

**Sensitivity towards Industry Safety Levels**

While throughout the previous analysis I used a safety level of 99.5% for the insurance company, I now also study the implications for 99.75% and 99.9%.\footnote{In 2011, Allianz disclosed the results of their internal risk capital model based on the 99.97% confidence level. See Allianz SE (2012).} In the same way, I set the bank’s minimum total capital ratio to 8%, 9.25%, and 10.5%. In contrast to the reference situation, I fix the minimum capital requirements from \( t = 1, \ldots, 10 \) and set the long-term equilibrium interest rate to \( \theta = 0.03 \).

As Figure 9 illustrates, the insurer’s portfolio weights vary with the safety level in the insurance industry. Interestingly, the insurer shifts more assets into risky asset classes as its regulatory safety level increases. Thus, Figures 9a and 9c depict a higher proportion of stocks and corporate bonds. At the same time, investments into government bonds decrease (Figure 9d). In other words, the already high pressure to fulfill the guarantees to policyholders is intensified through an increase in the regulatory safety level for insurance companies. In addition, I find that the insurer’s demand for bank bonds does not change for different insurance safety levels (Figure 9b).

When varying the bank’s safety level, the results are different. I find that although higher safety levels for the bank slightly reduce the insurer’s stock investment weight (Figure 9e), the insurer’s demand for bank bonds decreases substantially. This is in line with my previous finding, that shareholder value driven life insurers decrease their stake in bank debt in the long run due to the increasing safety level of banks, i.e., the decreasing return on bank bonds. As a result of the smaller demand for bank bonds, the insurer increases its corporate bond and government bond investments (Figures 9f through 9h).

Table 11 illustrates the resulting portfolio weights of the insurer for different combinations of safety levels in the banking and insurance sector. I find the largest proportion of corporate bonds when safety levels in both industries are highest, since in this case the demand for bank bonds is smallest.

Figure 10 depicts the insurer’s average return on assets resulting from the differences in the portfolio structure due to changes in the safety levels
Fig. 9. Life insurer’s portfolio weights under varying safety levels in the insurance industry with $k^B = 8.0\%$: (a) to (d). Varying safety levels in the banking sector with $(1 - \epsilon^I) = 99.5\%$: (e) to (h).
I find that the relatively small changes in the insurer’s portfolio structure resulting from a change in the safety level for insurance companies do not affect the insurer’s overall return on assets (Figure 10a).

In contrast, the insurer’s asset return varies with the bank’s minimum capital requirement. Since for higher safety levels in the banking sector the insurer mostly shifts investments towards government bonds, the resulting return on assets is comparably high in first years (Figure 10b).46 In the long

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**Figure 10.** Insurer’s return on assets and return on liabilities under varying safety levels in the insurance industry with $k^0 = 8.0\%$ ((a)) and varying safety levels in the banking sector with $(1 - \varepsilon^1) = 99.5\%$ ((b)).
In run, the insurer’s portfolio return is comparably smaller due to the large amount of low yielding government bonds in the portfolio. Nevertheless, the life insurer can avoid having to pay policyholders with equity capital.

When assessing the sensitivity of the insurer’s capital requirements to changes in the regulatory safety levels for banks and insurance companies, I find that they are mostly unaffected by the bank’s safety level, whereas they change with the insurer’s regulatory survival probability. Figure 11 illustrates the results.

As the safety level increases, the insurer has to hold more equity capital. In addition, the required equity capital increases faster and remains on a higher level. The same holds for various combinations of safety levels reported in Table 12. I measure the highest average capital requirement for the highest safety levels in both sectors (10.5%, 99.9%).

To analyze the contagion effects between bank and life insurer, I report the average conditional probability of contagion \( \pi^I_c \) from \( t = 1, \ldots, 10 \) in Table 13. I observe that \( \pi^I_c \) decreases with increasing safety levels in both the banking

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**Table 11. Average Portfolio Weights for Varying Safety Levels in the Banking and Insurance Industry**

<table>
<thead>
<tr>
<th>(1 - ( e^I ))</th>
<th>( k^I )</th>
<th>8.00%</th>
<th>9.25%</th>
<th>10.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.5%</td>
<td>( w_1 )</td>
<td>0.0298</td>
<td>0.0293</td>
<td>0.0287</td>
</tr>
<tr>
<td></td>
<td>( w_2 )</td>
<td>0.2109</td>
<td>0.1675</td>
<td>0.0806</td>
</tr>
<tr>
<td></td>
<td>( w_3 )</td>
<td>0.1720</td>
<td>0.1793</td>
<td>0.1894</td>
</tr>
<tr>
<td></td>
<td>( w_4 )</td>
<td>0.5873</td>
<td>0.6239</td>
<td>0.7013</td>
</tr>
<tr>
<td>99.75%</td>
<td>( w_1 )</td>
<td>0.0332</td>
<td>0.0325</td>
<td>0.0317</td>
</tr>
<tr>
<td></td>
<td>( w_2 )</td>
<td>0.2106</td>
<td>0.1634</td>
<td>0.0830</td>
</tr>
<tr>
<td></td>
<td>( w_3 )</td>
<td>0.1795</td>
<td>0.1861</td>
<td>0.1944</td>
</tr>
<tr>
<td></td>
<td>( w_4 )</td>
<td>0.5767</td>
<td>0.6180</td>
<td>0.6908</td>
</tr>
<tr>
<td>99.9%</td>
<td>( w_1 )</td>
<td>0.0345</td>
<td>0.0332</td>
<td>0.0321</td>
</tr>
<tr>
<td></td>
<td>( w_2 )</td>
<td>0.2359</td>
<td>0.1765</td>
<td>0.0830</td>
</tr>
<tr>
<td></td>
<td>( w_3 )</td>
<td>0.1617</td>
<td>0.1688</td>
<td>0.1798</td>
</tr>
<tr>
<td></td>
<td>( w_4 )</td>
<td>0.5679</td>
<td>0.6215</td>
<td>0.7051</td>
</tr>
</tbody>
</table>

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This results mainly from the premium on long-term government bonds compared to short(er) maturing corporate bonds and bank bonds.
and the insurance industry and is lowest in case of high capital requirements for banks as well as high regulatory survival probabilities for insurers (10.5%, 99.9%).

In this case, the high amount of required equity capital absorbs most of the losses incurred by bank defaults. In addition, due to the low return
on bank bonds that stems from the high capital requirement for banks, the proportion of bank bonds in the insurer’s asset portfolio is comparably small. Interestingly, when increasing the safety level for the insurer while the bank safety level is set to 8.00% or 9.25%, \(\pi_c \text{ decreases}\). Whereas the total probability of default decreases due to higher capital requirements (Table 12), the demand for bank bonds does not change by much (Table 11). Thus, the probability of a bank triggered default increases.

In contrast, when increasing the bank’s regulatory safety level to 10.5%, \(\pi_c \text{ decreases}\). As a result, contagion is explicitly driven by the insurer’s demand for bank bonds and implicitly driven by banks’ capital requirements.

The effects of an increase in the capital requirements for banks (\(k^B\)) as well as an increase in the insurer’s survival probability (\(1 - \varepsilon^I\)) on the insurer’s equity capital (\(E^I\)) and demand for stocks (\(\tilde{S}\)), bank bonds (\(\text{ideB}\)),

<table>
<thead>
<tr>
<th>(k^B)</th>
<th>99.5%</th>
<th>(1 - (\varepsilon^I))</th>
<th>99.75%</th>
<th>99.9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00%</td>
<td>0.1109</td>
<td>0.1217</td>
<td>0.1354</td>
<td></td>
</tr>
<tr>
<td>9.25%</td>
<td>0.1110</td>
<td>0.1217</td>
<td>0.1356</td>
<td></td>
</tr>
<tr>
<td>10.5%</td>
<td>0.1108</td>
<td>0.1214</td>
<td>0.1361</td>
<td></td>
</tr>
</tbody>
</table>

Table 12. Average Required Equity Capital Ratio to Maintain a \((1 - \varepsilon^I)\) Survival Probability for Different Safety Levels for Banks and Insurers

<table>
<thead>
<tr>
<th>(k^B)</th>
<th>99.5%</th>
<th>(1 - (\varepsilon^I))</th>
<th>99.75%</th>
<th>99.9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00%</td>
<td>0.0093</td>
<td>0.0106</td>
<td>0.0159</td>
<td></td>
</tr>
<tr>
<td>9.25%</td>
<td>0.0072</td>
<td>0.0090</td>
<td>0.0145</td>
<td></td>
</tr>
<tr>
<td>10.5%</td>
<td>0.0025</td>
<td>0.0023</td>
<td>0.0018</td>
<td></td>
</tr>
</tbody>
</table>

Table 13. Average Conditional Probability of Contagion Per Year for Different Safety Levels in the Banking and Insurance Industry
corporate bonds ($\tilde{CB}$), and government bonds ($\tilde{GB}$), as well as the conditional probability of contagion ($\tilde{\pi}_C$), are summarized in Table 4.

**DISCUSSION AND CONCLUSION**

The present analysis assesses the challenges European life insurers face in the current environment characterized by low interest rates, high guaranteed returns, and changes in the financial regulatory system.

To study the optimal risk policy of life insurance companies, I develop a contingent claim framework with a direct financial connection between a bank and a life insurer. Solvency requirements for both companies limit the default probability and set the minimum amount of equity capital, respectively.

The numerical results show that the current interest rate environment forces shareholder value driven life insurers to change their asset allocation. The analysis indicates that life insurers’ asset returns could fall below the average guaranteed interest rate thus putting policyholders’ accounts at risk. This problem is magnified through a more severe scenario of prolonged low interest rates. Since the return on life insurer’s assets tends to adjust quicker to low interest rates than the growth rate of the liabilities, which includes high return promises to policyholders, a low interest rate environment poses a serious threat to insurers’ solvability over the medium term.

I find that, under the model assumptions, life insurers’ portfolio composition will change significantly over the mid-term since they engage in more risky investments, especially bank bonds. Particularly when the gap between the return on assets and the average guaranteed return on policyholder’s accounts becomes smaller, life insurers’ appetite for risk increases. However, due to the increasing capital and liquidity requirements for banks under Basel III, life insurers’ demand for investments into bank bonds declines in the long run. In addition, the results show that life insurers need to increase the amount of equity capital to cope with the given economic conditions as well as the change in their asset allocation.

By varying banks’ and insurers’ safety levels, I find that life insurers’ stock investment increases with the insurer’s safety level. Surprisingly, the demand for investments into bank bonds decreases with rising minimum capital requirements for banks, whereas it is mostly unaffected by changes in the insurer’s safety level. As a result, the conditional probability of contagion is driven by the insurer’s demand for bank bonds, which itself depends on the bank’s minimum capital requirements.
The results of the analysis depend on both the calibration of the model and the necessary simplifications adopted. The insurer’s capital requirements depend on its simplified asset structure and indirectly on the stylized liability structure of the bank. A more complex structure could improve the analysis and add robustness to the results. In addition, the model relies on simplifying assumptions that considerably influence the final result. In general, the model is a reduced version of a bank and a life insurer’s balance sheet without product line diversification, group diversification, or reinsurance activities. Moreover, policyholders’ reactions are missing.

Nevertheless, the results have profound economic implications. Given the importance of the European insurance sector among institutional investors, I show that in its current form, Basel III and Solvency II could reduce the strength of the financial connection between banks and insurance companies in the long run. Against this background, I strongly encourage regulators and policy makers to integrate banking and insurance regulation to minimize unintended effects. As a first step, I recommend that policy makers force a closer communication among banking and insurance regulators.

I consider two areas for further fruitful research. First, a detailed survey of the investments of insurance companies connecting them to the banking industry will increase the transparency of contagion channels between banks and insurers, which allows for a more detailed analysis of contagion risks. Second, an extension of the model towards financial conglomerates can reveal further insights on the assessment of optimal group solvency levels by taking into account intra-group transactions and possible cross-pledging commitments.

REFERENCES


