Is Risk Taking Beneficial to the Insured and to Society?

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Abstract: In this article, we discuss whether, and when, risk taking is beneficial to the insured and to the society at large. We establish models utilizing stochastic optimal control theory and obtain the optimal levels of risk taking from perspectives of both the insured and the society. As part of the analysis, we discuss the relation between insurance and excessive risk taking. [Key words: risk taking, moral hazard, stochastic optimal control, welfare loss (gain), insurance.]

INTRODUCTION

As early as 1921, Frank Knight, in his seminal book (Knight, 1921), suggested that, in the long run, profits from risk taking could actually be negative. Knight's idea stands in stark contrast with much of the current thinking about risk. Current theories, especially theories of rates of return earned in financial markets, tend not to even consider such a possibility. Statistical evidence on the magnitude of profits derived from risk taking, while existing in some literature, can be difficult to evaluate, as the evidence often is subject to strong survivorship bias: Studies of successful risk taking in history involve entities that survived, but generally do not include consideration of entities that did not survive. For example, equity risk premium estimates are typically based on historical returns in US (or UK) markets, where market data are available as far back as the 19th century. But rarely

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are such estimates based on one of the largest equity markets in the world in 1900, the market of Russia, which did not survive at all after 1917.

We argue that it is important to distinguish between risk taking behavior that is beneficial from that which is harmful and, as a society, that we consider some intervention to encourage beneficial risk taking and limit harmful risk taking. With such an approach, profits from risk taking may well be positive. For example, rearranging economic structures with insurance contracts, it is possible to increase the benefits of risk taking. This is, we argue, the central purpose of insurance from a societal perspective.

Insurance is typically thought of as a contract designed to protect the insured. For example, people who buy natural disaster insurance will receive loss compensation when natural disaster strikes. In addition, the insured may have indirect gains from the contract. For example, an insured entity with fire insurance may enjoy a gain in its credit rating (Arrow, 1951; and Hardy, 1923). However, it is also well known that insurance can encourage risk taking. Such a situation is typically termed *moral hazard.* Shahar and Logue (2012) noted that insurance can destroy incentives to minimize risk. For example, deposit insurance will encourage insured banks to offer loans to riskier business prospects, and engage in riskier business in general.

The effect of insurance on the society as a whole is that insured businesses and individuals can assume risks they could not or would not previously take. Regulators generally will take measures to try to manage and/or limit the additional risk taking by insured entities, i.e., moral hazard. For example, banks are subject to capital regulation requirements and regulatory oversight of their lending practices. Moral hazard resulting from insurance is usually thought of as an undesirable byproduct that should be avoided. However, in some situations, taking risk may be beneficial to the insured and to the society as well. For example, people can save time, improve their productivity, and derive other benefits by driving a car significantly more often, because they buy car insurance; without insurance, driving a car would necessarily be a very risky activity (clearly, with insurance, driving is still risky physically, but less risky financially). An enterprise may get more profit by taking more risk in their business thanks to purchasing business insurance, such as business interruption insurance, or workers compensation. Alternatively, a business may lower their expenses, e.g., spend less on risk mitigation by purchasing insurance instead—although this is possible in the short-term only, assuming the insurer is unable to detect and price this behavior; persistent negligence of that nature, however, will almost certainly result in increased insurance premiums that exceed such savings in loss mitigation expenses, if insurance is priced efficiently.

People can take a job instead of staying home to take care of their children, planting vegetables in the garden, and living at subsistence level, because they buy life insurance. If they stay at home, chances of dying prematurely are minimized, while an intense work schedule, especially in high-risk jobs, or involving substantial travel, may increase their probability of dying. People who buy health insurance can potentially afford significantly more expensive health care than those without health care insurance, so their health can be improved and the quality of their lives can be increased and their lives can even be greatly extended. Insurance reduces the constraint of fear that hangs over human decision-making processes and helps people to open up new areas of actions that might be riskier, but hopefully also innovative, that might be difficult to engage in without insurance (Lin and Wang, 2015).

In this article, we will show by establishing models that risk-taking behavior in some cases is beneficial to the insured and to the society as well. But excessive risk taking[4] will not be beneficial to the insured and is likely to be harmful to the society. Let us define the terms "risk taking" and "excessive risk taking." Risk taking is an action or activity in which someone takes risks to achieve a benefit. Excessive risk taking is an action or activity in which someone takes more risk than he would otherwise take, in order to achieve a benefit, because the costs of risks are borne by a third party. We define moral hazard or excessive risk taking as risk taking that is harmful to the insurer/insured and/or to society. In other words, excessive risk taking comes at a cost that is absorbed by government or other parties that is not fully accounted for in the process of making the decision to take on the risk.

Let us note that one may mistakenly assume that this means that any insurance contract leads to excessive risk taking, because the costs of insured risks are borne by the insurance company when the risk materializes. Such perception misses the central feature of an insurance contract: that the costs of random risks, paid when such risks materialize, while borne at that time by the insurance company, are paid by the insured entities in the form of the insurance premium. Thus, these costs may be rearranged in their timing and amounts, but on the societal scale, the insured entities pay for their additional risk taking via the premiums. Insureds, on the whole, pay for that additional risk taking, and then some, as insurance companies must cover their expenses and profits.

[4]We use the terms "excessive risk taking" and "moral hazard" interchangeably in the paper.

LITERATURE REVIEW

Moral hazard has been an important topic in economics literature. Beginning with Arrow (1971) and Pauly (1968), economists discussed two partial solutions to the problem of moral hazard: (1) incomplete coverage against loss (through contract modification) and (2) observation by the insurer of the care taken to prevent loss (and modification of premium based on the data so gathered). Shavell (1979) studied how to determine exactly when an insurance policy represents a compromise between no coverage and full coverage in the case where the insurer does not observe care. He also analyzed the choice concerning the timing of observation of care, and showed that imperfect information about care is valuable.

Moral hazard is an especially important topic in health insurance research. A seminal case was the so-called *RAND Study*: The Health Insurance Experiment (HIE) (Manning et al., 1987), which was completed in 1982 and found over-consumption of health services if there is no cost sharing (or lower cost sharing) by the insured. Cost-sharing insurance contracts were designed in order to reduce insurance premiums and to act as a tool of risk management by creating incentives for the insured to lower the costs of claims. The study also found that moral hazard results in welfare loss. However, de Meza (1983) argued: "With rare exceptions the provision of actuarially fair health insurance tends to substantially increase the demand for medical care by redistributing income from the healthy to the sick. This suggests that previous studies which attribute all the extra demand for medical care to moral hazard effects may overestimate the efficient costs of health care."

Zweifel and Manning (2000, pp. 413–414) wrote: "From a normative point of view, moral hazard can be argued to cause a negative externality to the extent that it causes the insurer to increase premiums for everyone. Thus, moral hazard should be avoided. However, some amount of moral hazard may be deemed beneficial[5] for two reasons. First, to the extent that physicians wield a collective monopoly, the quality of medical care consumed falls short of the optimum. The increase in quantity caused by the moral hazard effect of health insurance can be efficiency-enhancing in this situation (Crew, 1969). Second, moral hazard may encourage the use of a more cost-effective medical service at the expense of a less cost-effective

[5]As described earlier, we define moral hazard or excessive risk taking as risk taking that is harmful to the insurer/insured and/or to the society. Therefore, the "moral hazard" in this statement, and in the statements by Nyman (2007) in the following paragraph, should be more accurately stated as "risk taking" based on our definition and discussion in this paper.

one within an insurance scheme (Pauly and Held, 1990). Thus, the optimal amount of moral hazard is positive rather than zero."

Nyman (2007) noted that moral hazard in health care insurance sometimes is not welfare loss but welfare gain. He states: "A large portion of moral hazard actually represents health care that ill consumers would not otherwise have access to without the income that is transferred to them through insurance. This is efficient and generates a welfare gain."

The studies described above only qualitatively emphasize that risk taking sometimes will generate a welfare gain. However, there is one extremely important problem remaining to be solved. That is, how do we determine the optimal risk taking level theoretically with the help of mathematical models?

In this paper, we try to fill this theoretical gap. We examine how to determine optimal risk taking of the agent by using stochastic optimal control theory. It also is important to note that we generally assume lawful and honest activities of both the insured and the insurer. In a real-world situation, the process may get complicated by undetected claim fraud, which clearly would be welfare-reducing.

MODELS

Analysis of the Critical Condition of Taking Risks with an Example of Automobile Insurance

Before establishing the stochastic optimal control model, let us first analyze the question of whether there can be a critical distinction between the insured taking a rational risk or creating a moral hazard, by using a simple example.[6] Assume that Mr. A has initial wealth of $12,000 and a car worth $4,000, and let us also assume that when an accident happens, the car will be completely destroyed (so that its value becomes zero). Assume that the probability of accident is 0.5 when Mr. A drives his car recklessly, while if he drives carefully the probability of accident is 0.2. Let us assume that the cost of driving carefully, and necessarily slower, is x (for example, opportunity cost of time). Assume that the utility function is the square root of wealth. We can get the rational price of $800 (i.e., $4,000×0.2) when driving carefully and $2,000 i.e., ($4,000×0.5) when driving recklessly (here,

[6]The example is based on an example created by Sun (2013, p. 33). In her text book, she assumes x =1000 dollars and reaches the conclusion that driving a car recklessly is always beneficial to the insured. Here, we provide a different discussion, subject to the condition that driving a car recklessly could be beneficial to the insured.

we neglect the loss to the third party). Then the critical condition for Mr. A to drive his car recklessly, based on expected utility comparison, is

$$0.8\sqrt{16000 - x - 800} + 0.2\sqrt{16000 - x - 800} < 0.5\sqrt{16000 - 2000} + 0.5\sqrt{16000 - 2000},$$

or $\sqrt{16000 - x - 800} < \sqrt{16000 - 2000}$, resulting in $x > 1400$. Thus, we see that driving recklessly is beneficial to the insured when the cost x is greater than \$1,400 dollars, and vice versa. Therefore, whether an insured is assuming rational risk, or creating a moral hazard, depends on whether the outcome is beneficial to him.

In the discussion above, we neglect the expected loss resulting from driving recklessly, such as the loss due to the increased risk of death or disability due to a traffic accident. However, in the following, we will discuss the optimal condition under which risk taking or moral hazard is beneficial to the insured, and to society, by stochastic optimal control theory. We will consider all benefits and losses resulting from risk taking / moral hazard.

Assumptions and Stochastic Differential Equation of Wealth

Assume that the insurance premium is *P*. It is important that the insurance premium P should not only include the risk of insured exposure but also include risk taking. Assume risk taking occurs with both the insurer and the insured. For example, an insurer will take risk in their investment portfolio after selling insurance. Assume that the amount of risky assets is π if there exists investment risk taking and the amount of risky assets is $\pi(1 + \alpha_1)$ if there exists excessive risk taking. That is, allocation to risky assets will increase by α_1 in the case of excessive risk taking. Meanwhile, the insured will take risks with their insured properties. Assume that the volatility of claim loss will increase by α_2 and the claim loss increase will be $B_3\alpha_2$ because of the risk taking of the insured after insurance, where B_3 is a constant.

Assume also that the increased premium due to risk taking is expressed as:

$$P_1(\alpha_2) = B_2\sigma_D\alpha_2, \tag{1}$$

where B_2 is a constant, and the premium for risk exposure without considering risk taking by the insured is P_2. Assume that the additional net benefit obtained by the insured because of taking risk is

$B_1\alpha_1\sigma + B_4\alpha_2\sigma_D$, where B_1 and B_4 are constants, and $B_4\alpha_2\sigma_D$ is the external benefit obtained by the insured due to his/her risk taking behavior, such as saving of time, or saving of risk management cost, minus the direct loss to the insured resulting from risk taking, such as the cost of death or disability. We assume that the additional net benefit obtained by the insured does not include any benefit obtained in violation of the law, or in violation of the insurance contract specifically; let us stress that there are no hidden benefits or costs not accounted for, which would be possible if conditions of contracts are not adhered to (as noted above, this is an assumption of lawful and honest activities of both the insured and the insurer). Then the net profit satisfies the following stochastic differential equation:

$$dX = (\pi(1+\alpha_1)(\mu - r) + X \cdot r - (P_1(\alpha_2) + P_2) + p_0 + (B_1\alpha_1\sigma + (B_3 + B_4)\alpha_2\sigma_D)dt + \pi(1+\alpha_1)\sigma dW_1 + (1+\alpha_2)\sigma_D dW_2) \quad (2)$$

where μ is the return rate of risky assets, r is risk-free interest rate, dW_1 is Geometric Brownian Motion with standard deviation σ, dW_2 is a diffusion process with diffusion coefficient σ_D, $\alpha_2\sigma_D$ is the risk not included in insurance that would take place, which might be due to the moral hazard created by the insured (for example, when the death or disability risk of the insured increases because the insured drives more quickly) or the loss risk of deposit insurance, which exceeds the upper claim limit, with increase due to the risk taking by the insured, and dW_1 and dW_2 are two independent stochastic processes.

HJB Equation and Optimal Solutions

We formulate the problem of maximizing the exponential expected utility of the terminal social wealth including the wealth of the insurer and the insured. Given initial values of time, t_0, the wealth of the insurer, X_0, the objective function over the class of admissible controls A_{t_0, X_0} is given by:

$$J((t_0 = 0, X_0 = x);(\pi,\alpha_1,\alpha_2)) = E_{t_0 = 0, X_0 = x}(U(X(T))). \quad (3)$$

The optimal problem can be expressed as: Find the value function $V(t,X)$ and optimal solutions of $(\pi,\alpha_1,\alpha_2) \in A_{t_0, X_0}$, which satisfy the condition:

$$V(t,X) = \sup_{(\pi,\alpha_1,\alpha_2) \in A_{t,X}} (J(t,X);A). \quad (4)$$

It is not difficult to show that $V(t,X)$ is a Markov process. For any twice continuously differentiable function $h \in C^{1,2}(O) \cap C(\overline{O})$, where $O{:} = (0,T) \times (0,\infty) \times \ldots (0,\infty)$ and $\overline{O}$ denotes the closure of O, there exists a partial differential operator $L^{\pi,\alpha_1,\alpha_2}[h(x)]$:

$$L^{\pi,\alpha_1,\alpha_2}(h(x,p)) = \frac{\partial h}{\partial t} + \Big(\pi(\mu - r)(1 + \alpha_1) + X \cdot r - (P_1(\alpha_2) + P_2) + \tag{5}$$

$$p_0 + (B_1\alpha_1\sigma + (B_3 + B_4)\alpha_2\sigma_D)\frac{\partial h}{\partial x} + \frac{1}{2}((1 + \alpha_2)^2\sigma_D^2 + (\pi(1 + \alpha_1))^2\sigma^2)\Big)\frac{\partial^2 h}{\partial x^2}$$

It is not difficult to get the following verification theorem (see Mataramvura and Øksendal, 2008).

Theorem 1: *Suppose that there exists a function* $\phi(t,x,p) \in C^{1,2}(O)\mathrm{I}C(\overline{O})$ *and a Markov control* $(\pi^*,\alpha_1{}^*,\alpha_2{}^*) \in A$ *such that*

1. $L^{\pi,\alpha_1,\alpha_2}(\phi(t,x)) \geq 0$ *for all* $(\pi,\alpha_1,\alpha_2) \in A$ *and* $(t,x,p) \in O$;

2. $L^{\pi,\alpha_1,\alpha_2}(\phi(t,x)) = 0$ for all $(t,x) \in O$;

3. for all $(\pi,\alpha_1,\alpha_2) \in A$: $\lim_{t \to T^-}\phi(t,x) = U(X^{\pi,\alpha_1,\alpha_2}(T))$;

Then $\phi(t,x) = V(t,x)$, *and* $(\pi^*, \alpha_1^*, \alpha_2^*)$ *is an optimal (Markov) control.*

In order to obtain the optimal value function $V(t,x)$ and the optimal control $(\pi^*, \alpha_1^*, \alpha_2^*)$, we only need to solve the following HJB equation:

$$\begin{cases} \sup\limits_{(\pi,\alpha_1,\alpha_2) \in A} L^{\pi,\alpha_1,\alpha_2}(V(t,x)) = 0, \\ V(t,x) = \dfrac{e^{-\gamma x}}{\gamma}. \end{cases} \tag{6}$$

To solve the above HJB equation, we use a trial function similar to that in Browne (1995); also see Mao et al. (2016) to find a solution of the following form:

$$\phi^1(t,x) = -\frac{1}{\gamma}e^{-\gamma(xe^{r(T-t)} + f(t))}, \tag{7}$$

where $f(t)$ is a undetermined function and $f(T) = 0$.

Substituting the above trial function into equation (5) yields:

$$L^{\pi,\alpha_1,\alpha_2}[\phi^1(t,x)] = \phi^1(t,x) \times \Big\{ f_t - (\pi(\mu - r)(1 + \alpha_1)) - \tag{8}$$

$$(P_1(\alpha_2) + P_2) + p_0 + (B_1\alpha_1\sigma + (B_3 + B_4)\alpha_2\sigma_D)\gamma e^{r(T-t)} +$$

$$\frac{1}{2}((1 + \alpha_2)^2\sigma_D^2 + (1 + \alpha_1)^2\pi^2\sigma^2)\gamma^2 e^{2r(T-t)}\Big\}$$

Putting $P_1 = B_2\sigma_D\alpha_2$ into equation (8) and maximizing over (π,α_1,α_2) yields the following first order conditions for the maximum point $(\hat{\pi},\hat{\alpha}_1,\hat{\alpha}_2)$:

$$-(\mu - r)(1 + \alpha_1) - B_1\alpha_1 + (1 + \alpha_1)^2\pi\sigma^2\gamma e^{r(T-t)} = 0 \tag{9}$$

$$-((\mu - r) + B_1) + (1 + \alpha_1)\pi\sigma^2\gamma e^{r(T-t)} = 0 \tag{10}$$

$$-(-B_2 + B_3 + B_4)\sigma_D + (1 + \alpha_2)\sigma_D^2\gamma e^{r(T-t)} = 0 . \tag{11}$$

By solving system equations of (9) and (10), we get:

$$\hat{\pi}(t)(1 + \alpha_1) = \frac{\mu - r}{\gamma\sigma^2 e^{r(T-t)}} \qquad \text{only when } B_1 = 0 . \tag{12}$$

Since taking excessive risk cannot make the agent better off, the net benefit obtained by the insurer is $B_1\alpha_1\sigma = 0$. We have:

$$\hat{\alpha}_1 = 0 . \tag{13}$$

Solving equation (11), we obtain:

$$\hat{\alpha}_2(t) = \frac{B_4 + B_3 - B_2}{\sigma_D\gamma e^{r(T-t)}} - 1 , \tag{14}$$

which means that optimal risk taking by the insured from the perspective of society satisfies equation (14). In other words, when the risk taking level by the insured satisfies:

$$\hat{\alpha}_2(t) \le \frac{B_4 + B_3 - B_2}{\sigma_D\gamma e^{r(T-t)}} - 1 ,$$

then risk taking by the insured is beneficial to the society, and vice versa. The function $f(t)$ is determined by the following differential equation:

$$f_t = \Big(\hat{\pi}(\mu - r) + (P_1(\hat{\alpha}_2) + P_2) - p_0 - \tag{15}$$

$$((B_3 + B_4)\hat{\alpha}_2\sigma_D)\gamma e^{r(T-t)} - \frac{1}{2}((1+\hat{\alpha}_2)^2\sigma_D^2 + \hat{\pi}^2\sigma^2)\gamma^2 e^{2r(T-t)}\Big).$$

Theorem 2: *When the expected utility function of the terminal social wealth is exponential, the optimal strategy* (π^*, α^*_2) *is given by equations of* (13) *and* (14), *and the optimal value function is:*

$$V(t,x) = -\frac{\exp(-\gamma x e^{r(T-t)} + f(t))}{\gamma}, \tag{16}$$

where $f(t)$ *is given by equation* (15).

Since the optimal risky assets after excessive risk taking is $\pi^*(1+\alpha_1) = \frac{\mu - r}{\sigma^2\gamma e^{r(T-t)}}$, which is exactly the same as that found by Merton (1971)—that is, $\pi^* = \frac{\mu - r}{\sigma^2\gamma e^{r(T-t)}}$ —and since $\hat{\alpha}_1 = 0$, the optimal strategy for the insurer is not to take extra risk in investment.

Therefore, our analysis shows that risk taking is good for the insurer (insured) and to the society as well. But extra risk taking cannot make the insurer (insured) better off and also it is not beneficial and even harmful to the society.

From the above equation $\pi^* = \frac{\mu - r}{\sigma^2\gamma e^{r(T-t)}}$, we know that the higher the return rate of risky assets (stock), the lower the volatility of the return rate of risky assets, and less risk aversion will encourage people to take more risk in their investment and vice versa.

Similarly, from the above equation $\alpha_2^*(t) = \frac{B_4 + B_3 - B_2}{\sigma_D\gamma e^{r(T-t)}} - 1$, we know that smaller underwriting risk and less risk aversion will encourage people to take more risk in their insured activity and vice versa.

In the following, we will discuss whether and under what condition it is optimal and beneficial for the insured to engage in risk taking for underwriting risk under fair pricing.

The Case of Fair Pricing

Without Consideration of the Government's Intervention

From equation (14), we know that there exists optimal level of risk taking, $\alpha_2{*}$. When $\alpha_2 < \alpha_2{*}, \frac{\partial E(U(X))}{\partial \alpha_2} > 0$, and increased risk taking is beneficial. When $\alpha_2 < \alpha_2{*}, \frac{\partial E(U(X))}{\partial \alpha_2} < 0$, and the situation is reversed, i.e., increased risk taking is harmful. From equation (14), we also know that if $B_4 + B_3 - B_2 > \sigma_D \gamma e^{r(T-t)}$, that is, $\alpha_2{*} > 0$, risk taking is beneficial to the insured. If pricing is rational, that is, if $B_3 = B_2$, the optimal condition for risk taking to be beneficial to the insured is:

$$B_4 > \sigma_D \gamma e^{r(T-t)} . \tag{16}$$

Or, stated another way:

$$\alpha_2{*} = \frac{B_4}{\sigma_D \gamma e^{r(T-t)}} - 1 > 0 \tag{17}$$

It is important to notice that if enough insureds are irrational and take excessive risk—that is, if continuous welfare loss exists ($B_4 < \sigma_D \gamma e^{r(T-t)}$) — the market would not sustain itself in the long run without some form of intervention to avoid excessive risk taking. In the following, we will discuss the optimal condition that it is beneficial to the society by taking risk under the condition that the government undertakes some intervention.

From inequality (16), we find that under conditions of smaller volatility of risk exposure, less risk aversion, and lower risk-free interest rate, risk taking may possibly benefit the society. And higher benefit obtained by the insured from risk taking will also benefit the society. However, the benefit obtained by the insured from risk taking depends on economic efficiency of the society, productivity level, and capital stock. Generally speaking, if you can produce more or if you have a lot of wealth you can take more risk. And the risk-bearing capability of the society also depends on productivity and capital. From equation (16), we can show that the optimal level of risk taking $\alpha_2{*}$ is an increasing function of B_4, which is the net benefit obtained by the insured from risk taking. Also, culture matters, because, other things being equal, if other people are more likely to be honest with you, or help you when you have problems, you can take more risk.

With Consideration of the Government's Intervention

If the government does not incur any costs when the insured obtains the benefit of B_4, then $B_4 > \sigma_D \gamma e^{r(T-t)}$ is also the condition that risk taking is beneficial to the society. If the government incurs costs of $B_5 \alpha_2 \sigma_D$ for the behavior of risk taking of the insured (for example, the cost resulting from overconsumption of health services or subsidies),[7] and the benefit to the insured is increased to $B_4(1+\delta)\alpha_2\sigma_D$, then the optimal condition for risk taking to be beneficial to the insured is $B_4(1+\delta) - B_5 > \sigma_D \gamma e^{r(T-t)}$. Meanwhile, the optimal level of risk taking increases to

$$\hat{\alpha}_2(t) = \frac{B_4(1+\delta) - B_5}{\sigma_D \gamma e^{r(T-t)}} - 1. \tag{18}$$

Therefore, if the increased subsidy by government B_5 is less than $B_4\delta$, it would encourage the insured to take more risk. If the benefit obtained by the insured has an externality, that is, if it increases benefit of $\delta_1 \sigma_D \alpha_2$ to the society (such as improved health of the labor force because of the subsidy in health care insurance), then, when $B_4(1+\delta+\delta_1) > B_5$, the optimal condition for risk taking to be beneficial to the society is:

$$B_4(1+\delta+\delta_1) - B_5 > \sigma_D \gamma e^{r(T-t)}. \tag{19}$$

Although subsidies will encourage the insured to take more risk, if the benefit obtained by the insured and by society is greater than the subsidies, it is still beneficial to the society. However, if:

$$B_4(1+\delta+\delta_1) - B_5 < \sigma_D \gamma e^{r(T-t)}, \tag{20}$$

or

$$\alpha_{2s}^*(t) = \frac{B_4(1+\delta+\delta_1) - B_5}{\sigma_D \gamma e^{r(T-t)}} - 1 \le 0, \tag{21}$$

[7]When considering the cost (e.g., spending on subsidies) assumed by the government for additional risk taking or moral hazard of the insured, the total wealth should include any net benefit resulting from subsidies.

then risk taking is not beneficial, or is even harmful to the insured and to the society. In this situation, decreasing risk through risk management can benefit the insured and the society. Let the cost of risk management be $B_6\alpha_2\sigma_D$, and the benefit obtained or loss suffered be $B_7\sigma_D|\alpha_2|$ then the optimal condition for risk management to be beneficial to the insured, or to the society, is:

$$-B_6 + B_7 < \sigma_D\gamma e^{r(T-t)}. \tag{22}$$

Or, equivalently,

$$\alpha_{2s}^*(t) = \frac{-B_6 + B_7}{\sigma_D\gamma e^{r(T-t)}} - 1 \le 0. \tag{23}$$

Here we assume that the claim risk is fairly priced.

The Case of Irrational Pricing

Let us discuss the condition for risk taking to be beneficial to the insured and to the society under the condition of irrational pricing. On one hand, if risk taking is underpriced, that is, when $B_3 - B_2 > 0$ —that is, the insurance is too cheap—the condition for risk taking to be beneficial to the insured is

$$B_4 + B_3 - B_2 > \sigma_D\gamma e^{r(T-t)}. \tag{24}$$

Or, equivalently,

$$\alpha_{2s}^*(t) = \frac{B_4 + B_3 - B_2}{\sigma_D\gamma e^{r(T-t)}} - 1 > 0. \tag{25}$$

Comparing inequality (25) with inequality (17), we see that the insured would take more risk in the case of underpricing. The net benefit obtained by the insured due to taking risk $B_4\alpha_2^*\sigma_D$ will be greater, if $B_4 > 0$ due to underpricing. However, equation (25) shows that the optimal value of α_2^* will be smaller if $B_4 < 0$. Therefore, the insured will take less risk in the situation of $B_4 < 0$ even under the situation of underpricing.

Since the insurance company will pay extra claim loss of $B_3 - B_2$, the condition for risk taking to be beneficial to the society is:

$$B_4 + B_3 - B_2 - (B_3 - B_2) = B_4 > \sigma_D\gamma e^{r(T-t)}, \tag{26}$$

and optimal value of

$$\alpha_{2s}^* = \frac{B_4}{\sigma_D \gamma e^{r(T-t)}} - 1 , \tag{27}$$

where α_{2s}^* is the optimal coefficient of risk taking (moral hazard) for underwriting risk for society. The net benefit obtained by the society due to taking risk, $B_4 \alpha_{2s}^* \sigma_D$, will be the same as that under the situation of fair pricing.

If $B_4 < 0$, however, the optimal level of risk taking will be negative (the net benefit obtained by the insured due to risk taking is negative). That is, the insured will take even less risk than in the situation where $B_4 > 0$.

Comparing equation (25) with equation (27), we find that the optimal risk taking level for the society, α_{2s}^*, is smaller than that for the insured under the situation of underpricing. Therefore, price regulation might be considered to discourage the risk taking (moral hazard) of the insured.

On the other hand, if the risk taking is overpriced, that is, when $B_3 - B_2 < 0$ —that is, the insurance is too expensive—the condition for risk taking to be beneficial to the insured is the same as in the case of underpricing (see equation (25)). However, the insured would take less risk since the value of α_2^* is smaller due to $B_3 - B_2 < 0$.

The optimal risk-taking coefficient of the society, α_2^*, is the same as that under the case of underpricing, since the benefit gained by the insurer, $B_2 - B_3$, is the same as the loss (overpayment) by the insured due to overpricing.[8] In contrast with the situation of underpricing, the optimal risk taking level for the society, α_{2s}^*, is greater than that for the insured under the situation of overpricing. In this case, price regulation might restrict overpricing by insurers to encourage more risk taking. If considering the cost assumed by the government because of risk taking of the insured, the analysis is similar to the case under rational pricing. Hence, we will not repeat that analysis.

SUMMARY AND CONCLUSIONS

In this article, we discuss if and under what conditions risk taking can be beneficial to the insured and to the society. We establish models based on stochastic optimal control theory, and our optimization objective is to maximize exponential expected utility of the terminal social wealth. Based

[8]Here we assume that the social benefit is the sum of the benefit obtained by the insured and the insurers.

on and subject to the above, we find optimal levels of risk taking. We discuss the optimal conditions for risk taking to be beneficial to the insured and to the society respectively. We also discuss insurance, moral hazard, and their relation to each other by using automobile insurance as an example.

Results of our analysis indicate that risk taking is, under certain conditions, beneficial, but in other conditions it is detrimental to society. In a sense, decisions on individual risk taking are economic (real economic) projects. The central issue for society is whether the particular project has positive or negative economic value. The cost of risk for the project should be properly calculated. If the cost of risk is underestimated, poor economic projects will be funded. If the cost of risk is overestimated, fewer economic projects will be funded, and funding fewer projects could actually be welfare-decreasing for society if the projects that are not undertaken could have been profitable and valuable. Therefore, regulators charged with considering insurance prices should consider the potential effects of underpricing or overpricing risk in terms of efficient risk taking to maximize social outcomes. Therefore, the paper then can be viewed as guidance for public policy and government involvement, with the purpose of making insurance more welfare-enhancing.

Our paper, however, does not provide specific policy prescriptions, because optimality results depend on a variety of model parameters. We mainly propose that regulatory actions should not seek elimination of moral hazard, or encouragement of it by subsidies for insurance coverage, but, rather, should seek efficient pricing of it, with consideration for all parameters affecting optimality. This may be a complex solution to implement practically, but it does provide a different direction to existing policies, which too often are either seeking to prohibit moral hazard or disregard its cost for the sake of expanding coverage.

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